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NANO-GEOMETRY

detection of ‘unknown’ pattern

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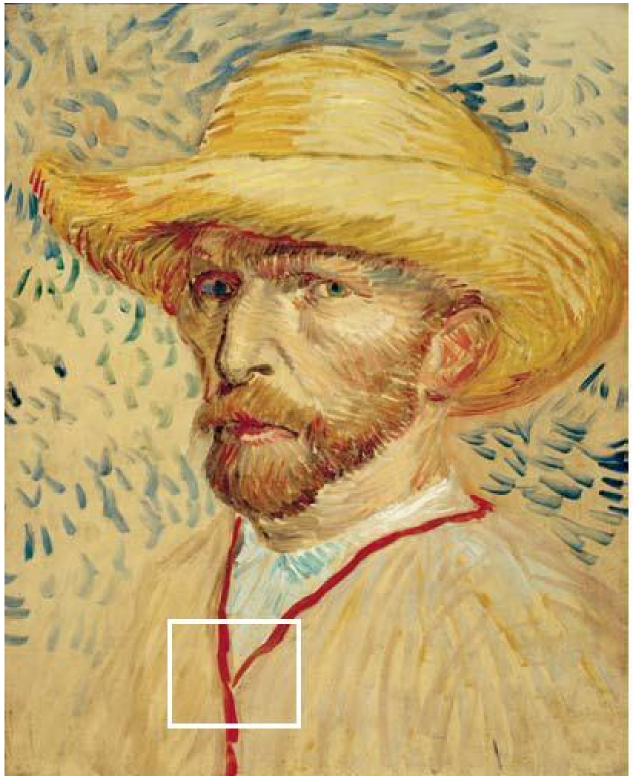
My lecture is dedicated to 75^{th} birthday of
Prof. Grozio Stanilov

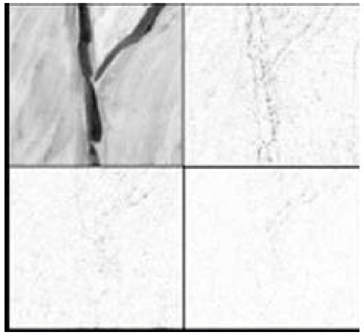
I will talk about

COMPUTER METHODS
IN THE SCIENCE AND IN THE EDUCATION

combined with geometry.
Motivation is given

- **by the close connection to material science:**
  Different kinds of steel **are** austenite **and** martensite:
  „is a given steel austenite or martensite?“
- **by the problem of detecting the origin of paintings:**
  Just at present neural network is used as a new tool for solving the question:
  „was a given antique originally produced by van Gogh?“





screening of a small part of the painting:

1. **from color to steps of gray**
2. **from steps of gray to „color 0“ or „color 1“**

Taylor, R.P., Micolich, A.P. and Jonas, D.: „Fractal analysis of Pollock’s drip paintings“. Nature 399 (1999), 422

0 abstract

**Some properties of metal can be detected
by analyzing the geometry of a polished cut image.**

**During the solidification of steel „lath martensite“ can appear,
and this kind of steel can be seen by the pattern of a polished cut:**

**„lath martensite is made up of lances,
which form packages of close parallel lances ...“**

(wikipedia)

**This formulation
and the collaboration with material scientists
gave a stimulation to this lecture,
which will show**

a geometrical and a statistical

description of the „direction“ of a lance.

**Such a definition is necessary for an automatic analyse
to detect „lath martensite“.**

detection of unknown pattern in material science (nano geometry)

1 structure of martensite steel

2 definition of „elliptic spot“

3 statistical solutions

1 structure of martensite steel

1.1 martensite steel (properties)

Only a small part of the produced steel is used for martensite steel (german metallurgist Adolf Martens (1850-1914)), which is

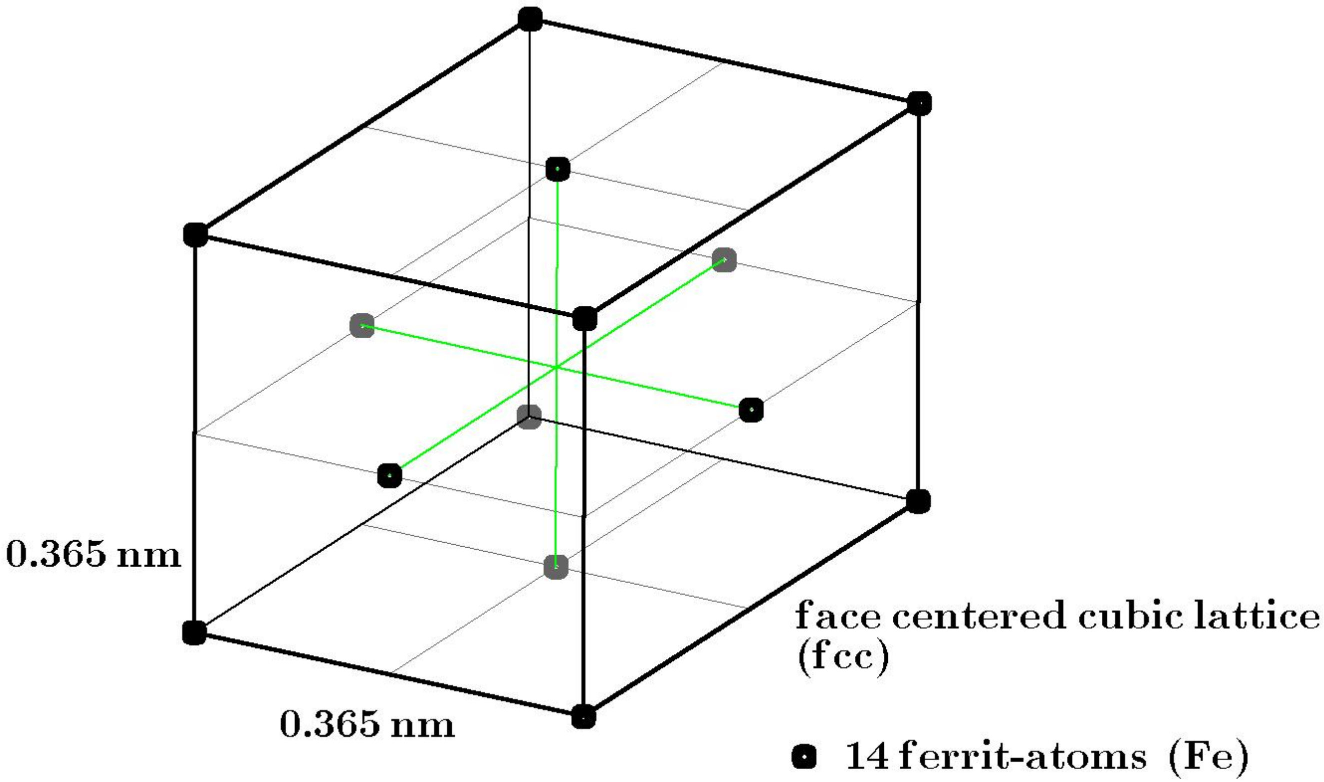
- **high-strength** (hochfest), **and**
- **highly wear-resistant** (hochverschleißfest)

and is a necessary material for cutting tools.

Here we will introduce an „austenite-unit“ as a system of 14 ferrit atoms which represent a face-centered cubic lattice (kubisch-flächen-zentriertes Gitter)
But we are looking for martensite!

If we accelerate the cooling process we can sometimes get martensi-te which is represented by body-centered cubic lattices (kubisch-raumzentriertes Gitter).

1.2 ferrit lattice with an (imaginary) carbon-atom outside



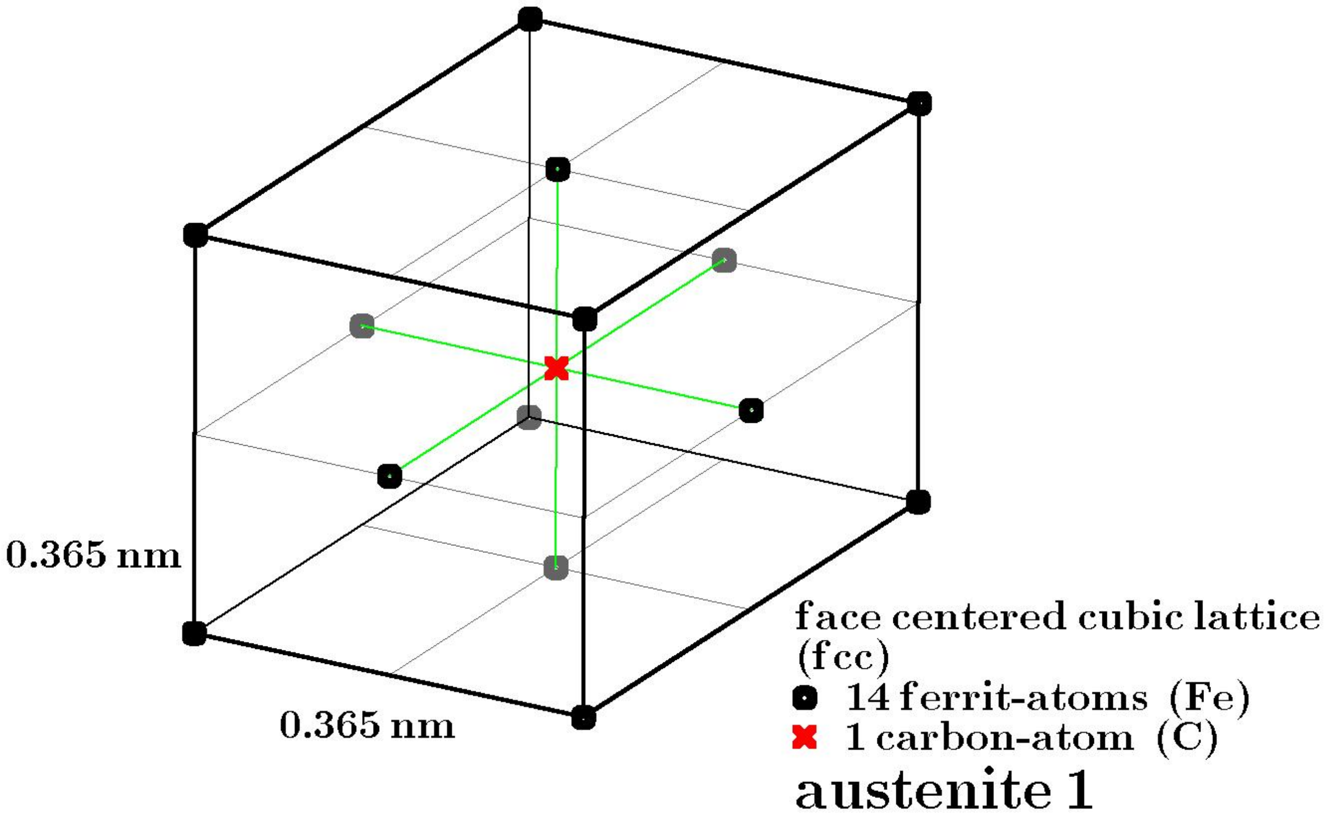
picture 1.1: ferrit unit (14 ferrit atoms)

atomic weight of Fe: 56
atomic weight of C: 12

atomic diameter of Fe: 0.30 nm
atomic diameter of C: 0.14 nm

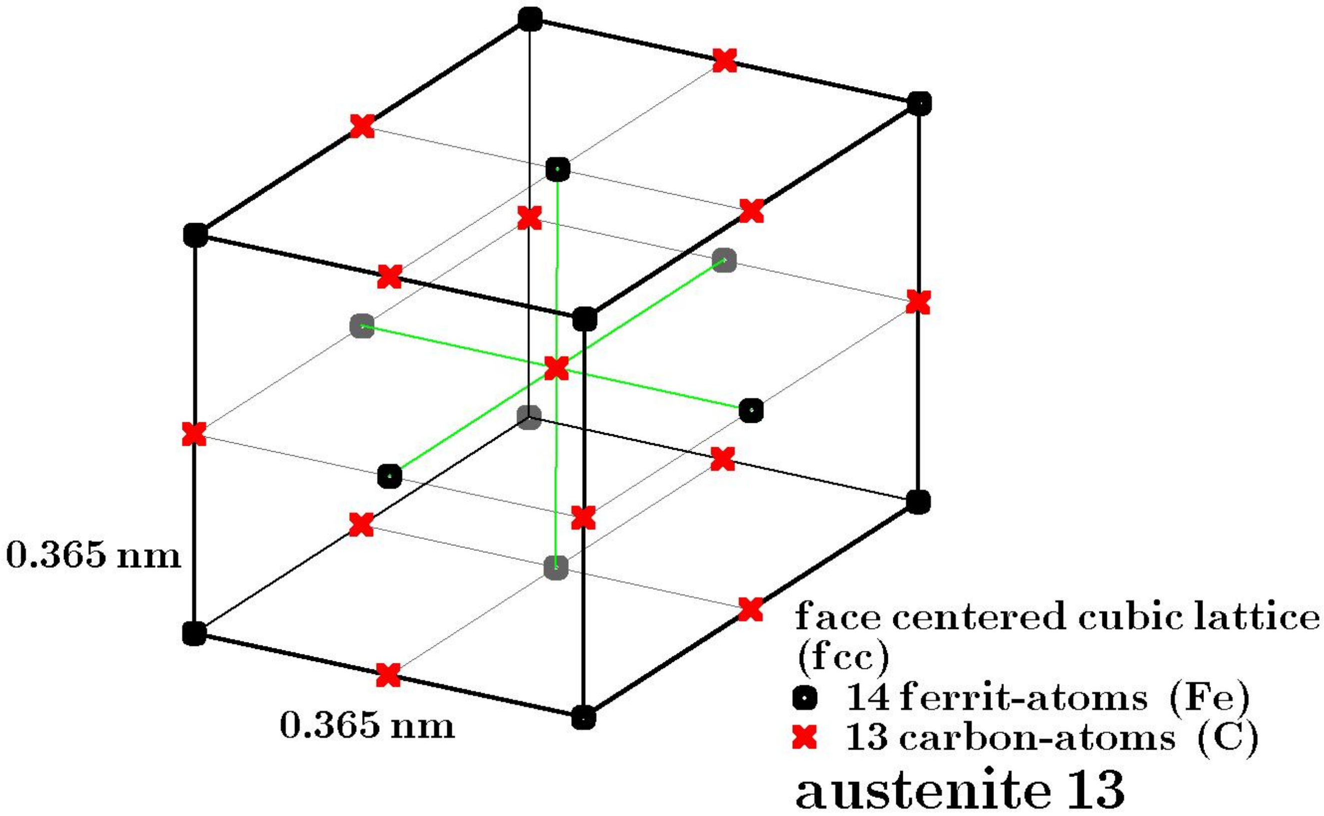
concentration of C: 0.00 %

1.3 austenite lattice (one carbon-atom inside of a ferrit-unit)



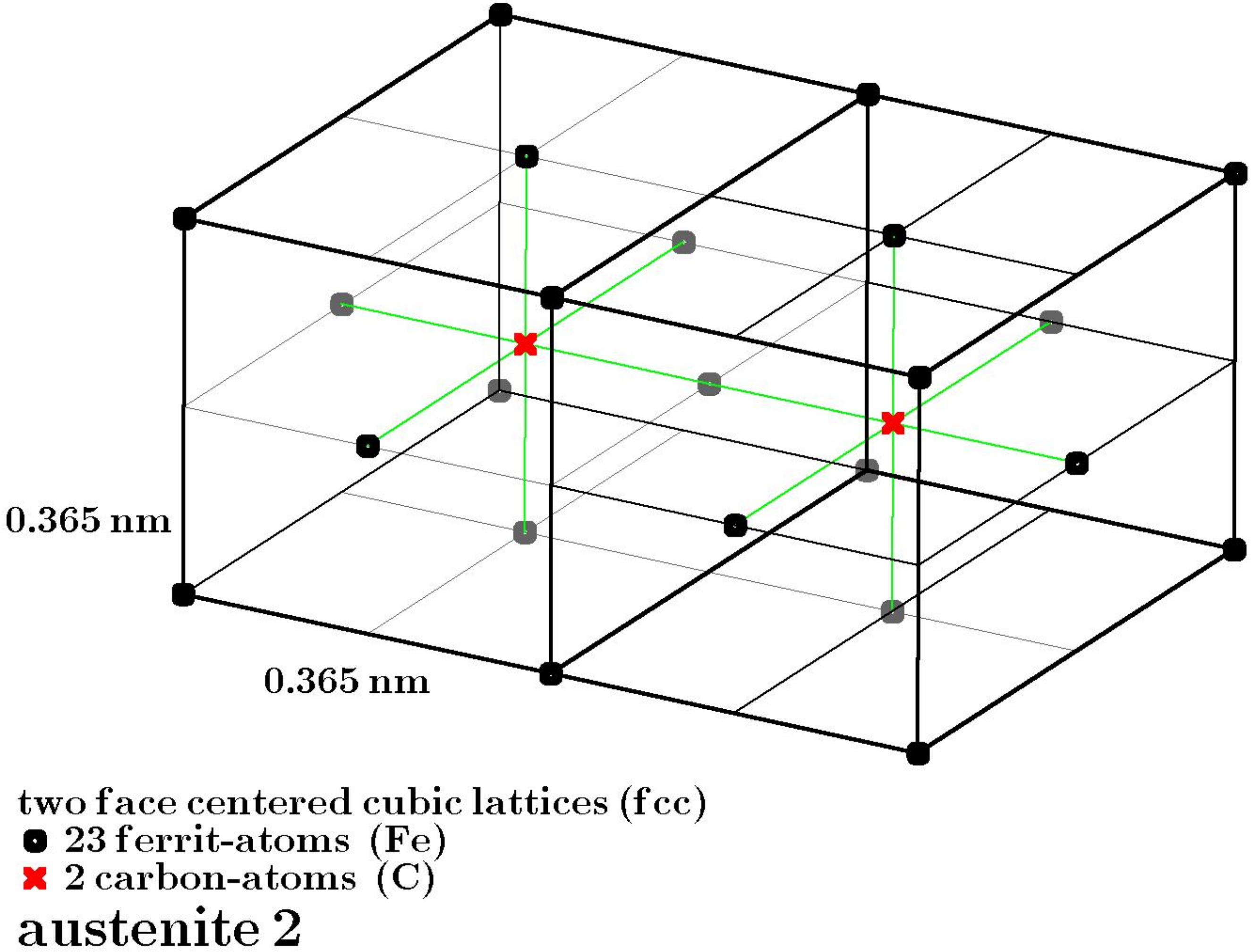
picture 1.2: **austenite: ferrit with 14 ferrit atoms and 1 carbon atom**
atomic weight of *Fe*: 56
atomic weight of *C*: 12
concentration of *C*: 1.50 %

1.4 austenite lattice (13 carbon inside of a ferrit-unit)



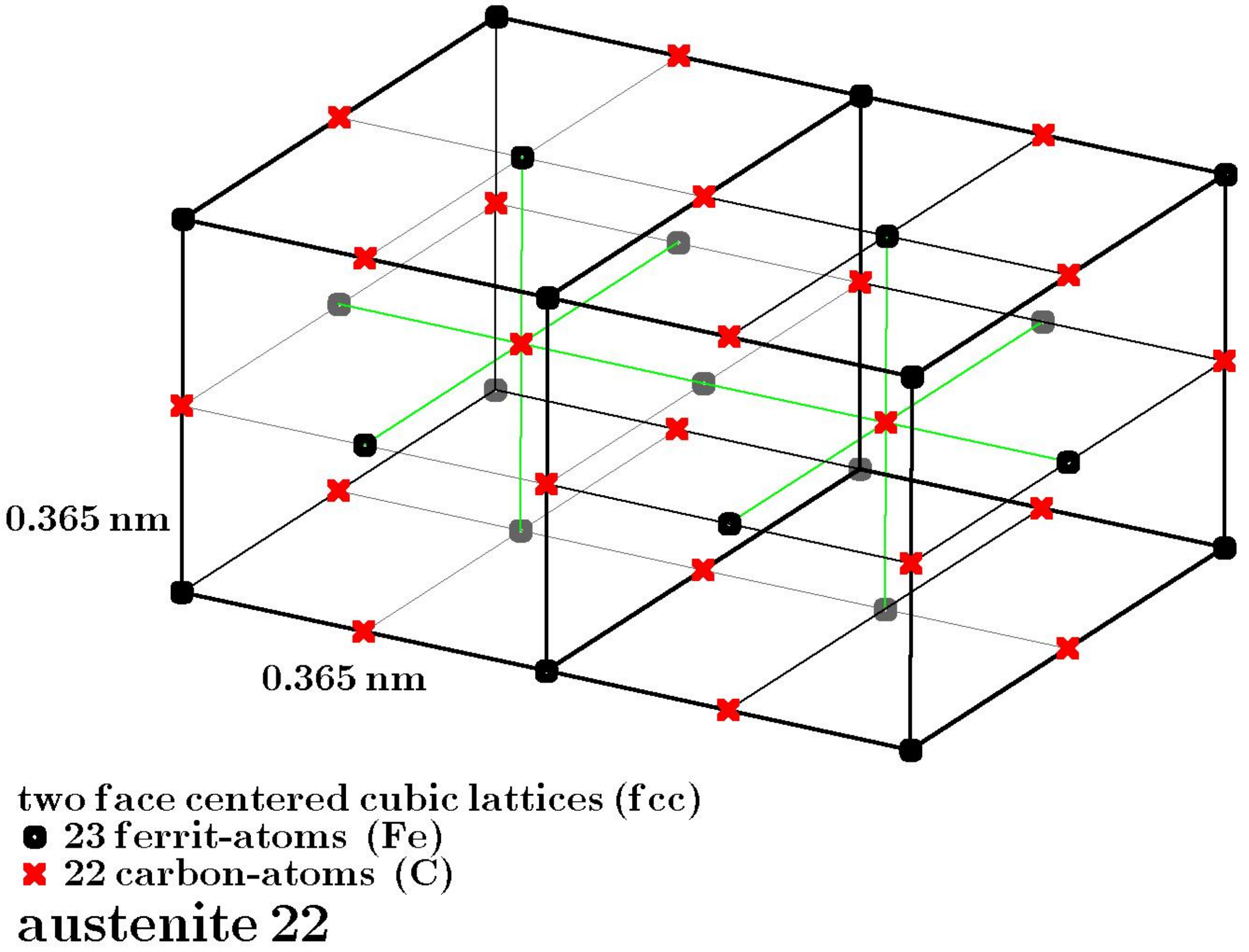
picture 1.3: **ferrit with 14 ferrit atoms and 13 carbon atoms**
atomic weight of *Fe*: 56
atomic weight of *C*: 12
concentration of *C*: 16.60 %

1.5 two ferrit units with 2% carbon



picture 1.4: **ferrit with 23 ferrit atoms and 2 carbon atoms**
concentration of C: 1.83 %

1.6 two ferrit units with 17% carbon



picture 1.5: **ferrit with 23 ferrit atoms and 23 carbon atoms**
concentration of *C*: 17.01 %

During solidification from the liquid phase of ferrit (or austenite) we find between 1392°C down to 911°C a *fcc*-lattice as we considered before:

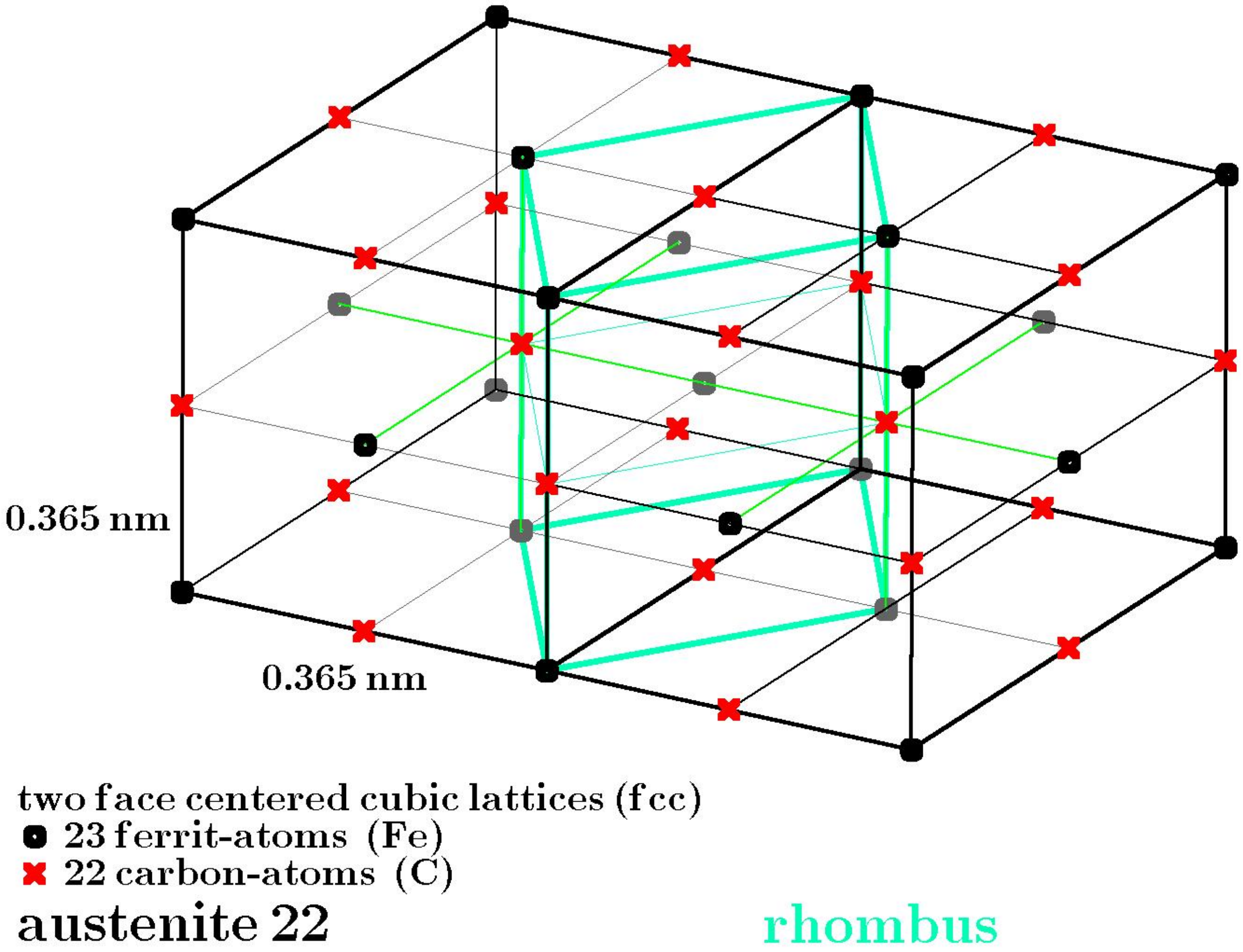
atom diameter of *Fe*: $D = 0.25\,nm$
atom diameter of *C*: $D = 0.15\,nm$
edge length of a austenite-unit: $a_{fcc} = 0.365\,nm$

On the edges of a unit the atoms do not touch each other, because there remains an interval $a_{fcc} - D = 0.11\,nm$.
In our last picture we considered two ferrit units (total 17% carbon), and we will look if it is possible to model the change from *fcc* to *bcc*:
The change to martensite appears in a very short time:
There is no possibility to diffusion, and therefore no exchange of atoms will happen: only a common motion of groups of atoms seems to be possible.

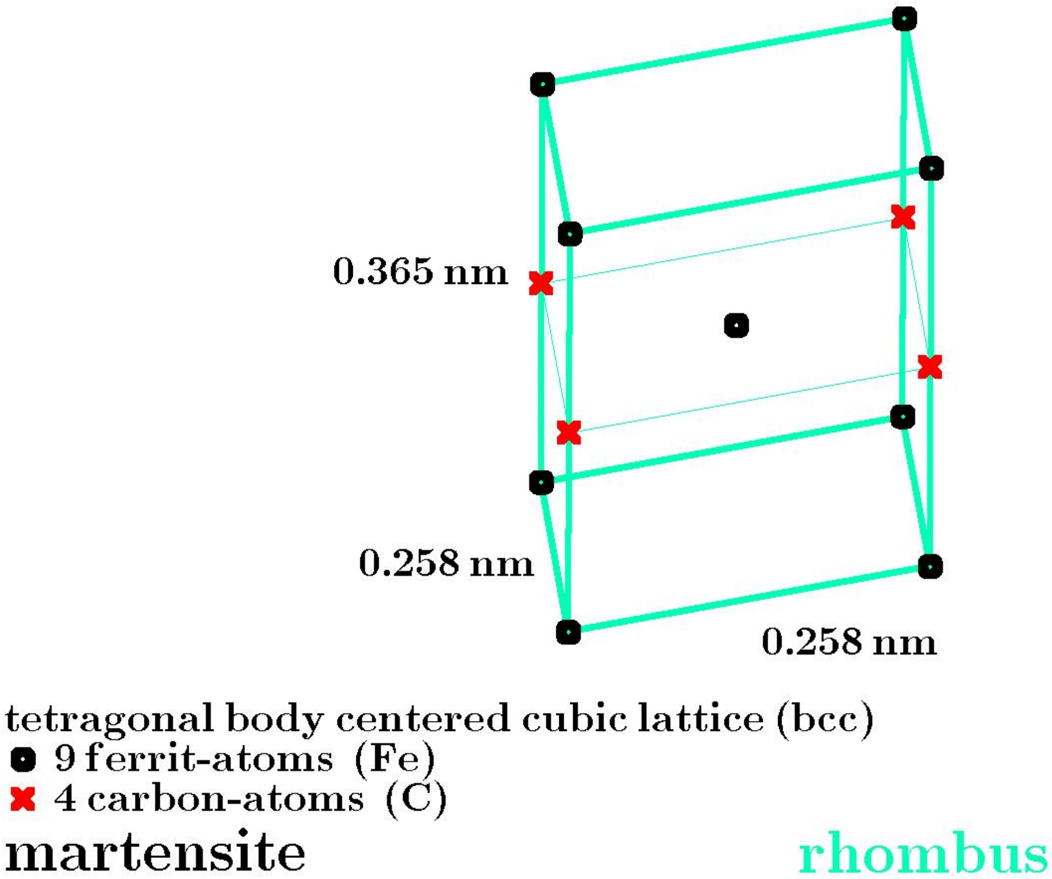
$\boxed{\textbf{We have to find a group of atoms which can move to a } bcc\textbf{-unit}}$

edge length of a *bcc*-unit (martensite): $a_{bcc,z} = 0.297\,nm$ **in z-direction**
edge length of a *bcc*-unit (martensite): $a_{bcc,x} = 0.286\,nm$ **in x,y-direction**

1.7 preparation for changing from *fcc* to *bcc*



picture 1.6: **using the system of 23 ferrit and 23 carbon atoms from *picture 1.5***
we will find a rhombus **inside - as a group of atoms which can move.**



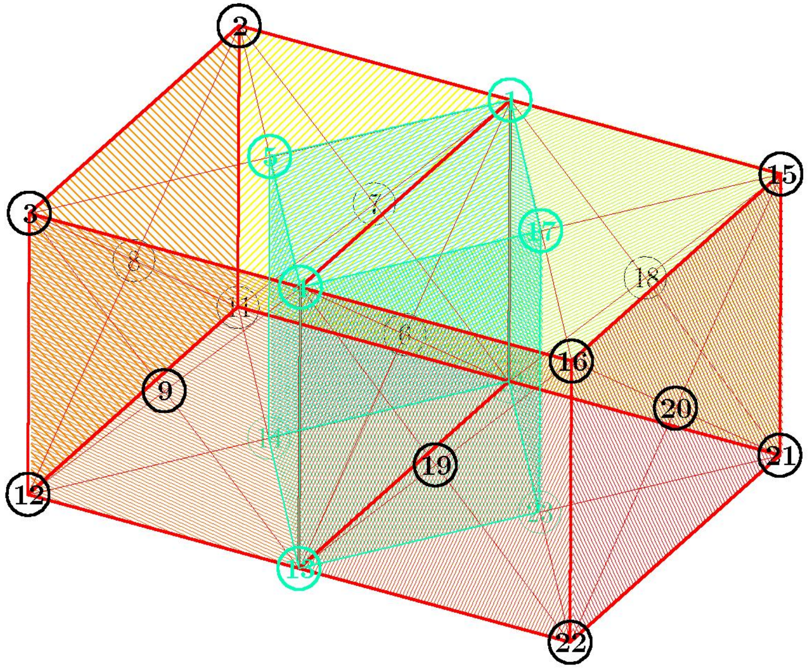
picture 1.7: **rhombus from *picture 1.6***

the dimensions have to change during parts of a second:

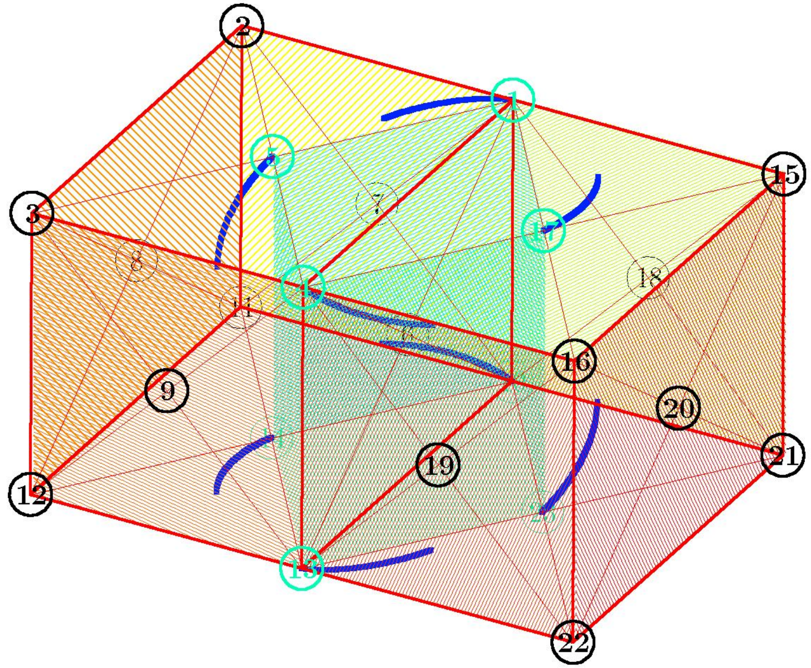
in z-direction: **from** $a_{fcc} = 0.365\,nm$ **to** $a_{bcc,z} = 0.297\,nm$ **in z-direction**

in x- and y-direction: **from** $0.258\,nm$ **to** $a_{bcc,x} = 0.286\,nm$ **in x, y-direction**

1.8 from two *fcc*-units to one *bcc*-unit



picture 1.8:
two austenite *fcc*-units
with „virtual lattice"
edge length $a_A = 0.365\,nm$,
atom diameter $0.25\,nm$



picture 1.9:
the „virtual tetragonal *bcc*-unit"
has to center with $45°$,
to shrink in $z-$direction
and to increase in $x-$ and $y-$direction

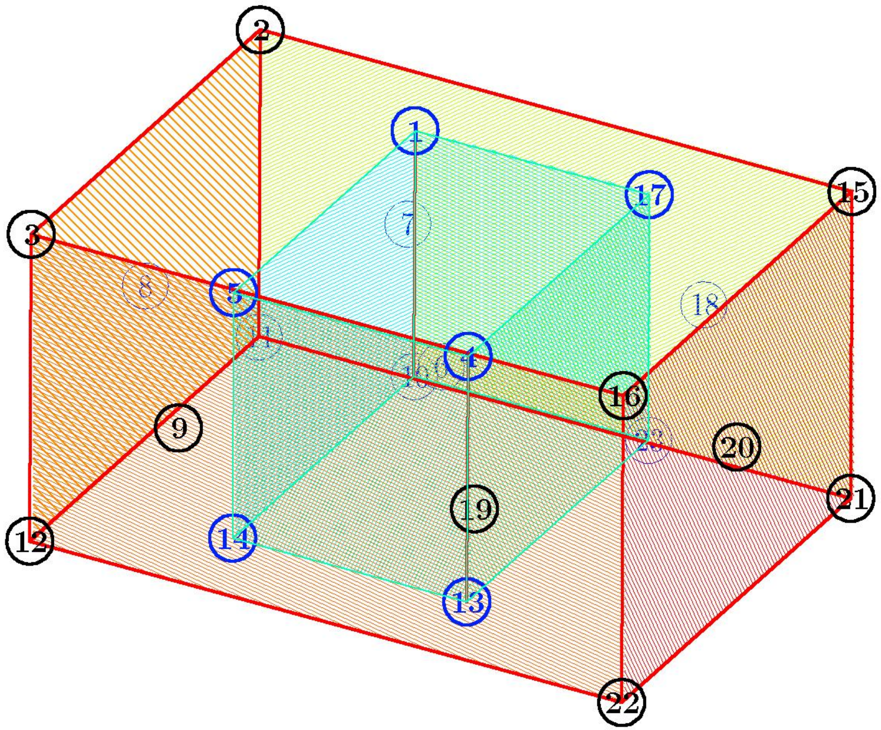
Deformation from austenit to martensite means:

- **center with** 45°
- **shrink in** $z-$**direction from** $a_{fcc} = 0.365\,nm$ **to** $a_{bcc,z} = 0.297\,nm$ **in** z**-direction**
- **increase in** $x-$ **and** $y-$**direction from** $0.258\,nm$ **to** $a_{bcc,x} = 0.286\,nm$

and then we will get „martensite“.

Remark: volume of a martensite-unit:

$$\boxed{v_{mart} = 0.286^2 \cdot 0.365 = 0.03\,nm^3 = 3 \cdot 10^{11}\,\mu\,m^3}$$



picture 1.10:

„virtual tetragonal body-centered lattice“ martensite

But consider: a lance of martensite of thickness $h = 0.5\,\mu\,m$ **has an enlargement of at least** $v_{lance} = 0.5^2 \times 3.5\,\mu\,m^3 = 0.875\,\mu\,m^3$ **- therefore such a lance consists of**

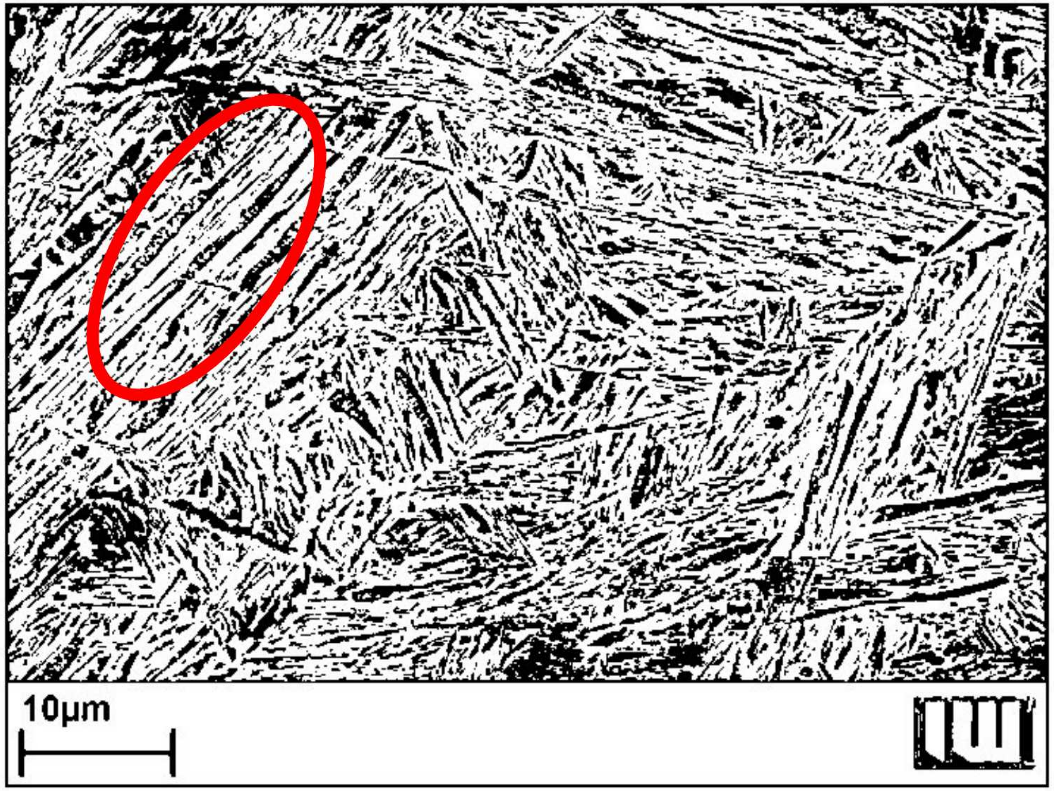
$$\boxed{\textbf{more than } 3 \cdot 10^{10} \textbf{ martensit-units.}}$$

2 definition of „elliptic spot“

2.1 problem

By etching a cut of (martensite) steel we can get a „polished cut image“, and we can choose a coordinate-system in a„natural way“.
We have to see

„packages of close parallel lances which are smaller than 1 $\mu\, m$“ (wikipedia)

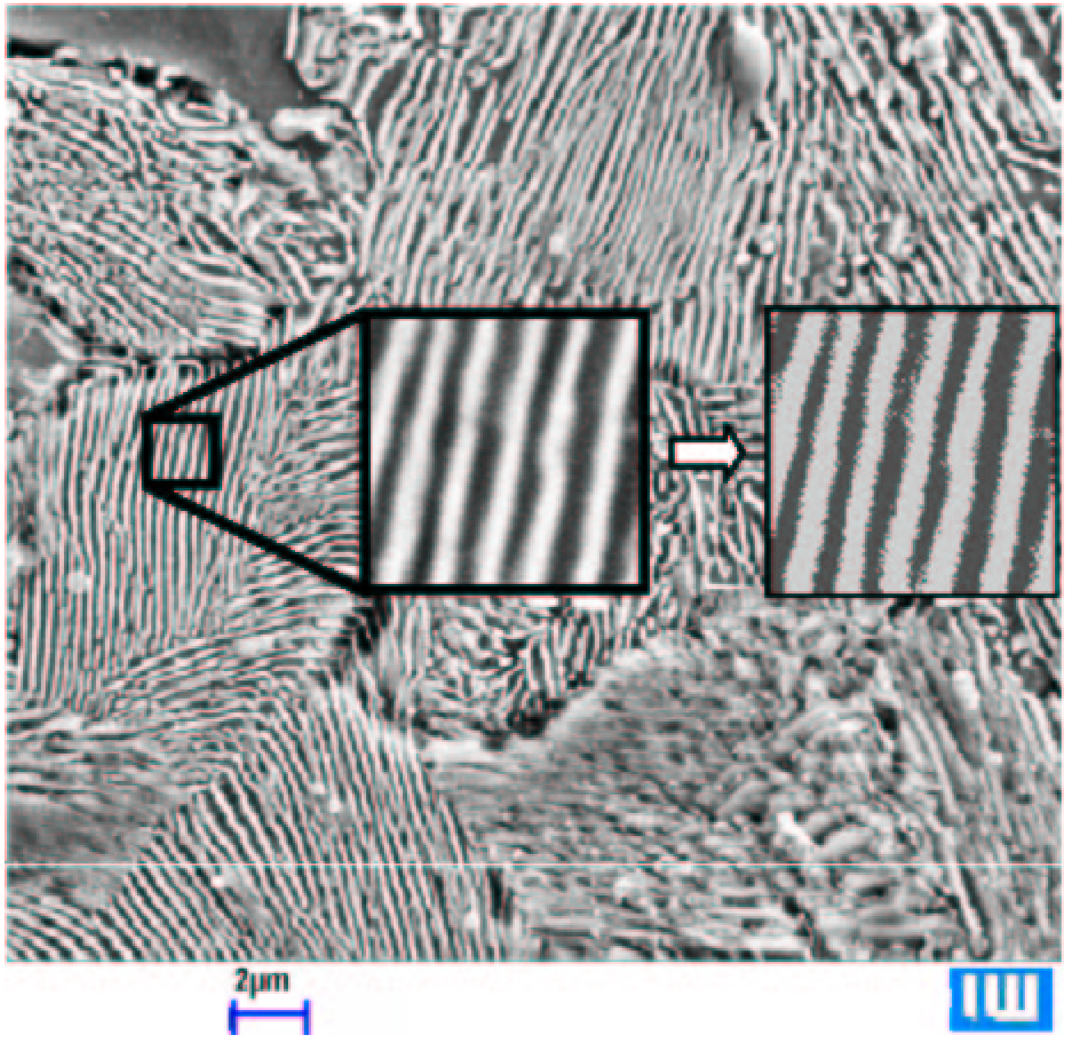


picture 2.1: **selected grains of martensite steel**

Nevertheless, we see

„packages $A(\alpha, n)$ of n close parallel lances with direction angle α“

In my lecture last year in Varna I gave some definitions and instructions for detecting a direction angle:



But at first we have to describe this picture; therefore we have to define a coordinate system of a *nano geometry* with *pixels*.

remark: In *picture 2.1* we have not only black and white colors but also some levels of grey, which we will attach to 0 or 1.

picture 2.2:
rough discretization of *picture 2.1*

From this rough discretization („screening“)
1 pixel $\approx 200\,nm \times 200\,nm$
we get a *nano geometry*
which has black and white points only.

Here we try to find

packages $A(\alpha, n)$ ***of*** n ***close parallel lances of black color***
with direction angle α

in a finite cooordinate system.

Then we has to ask if these packages define a martensite steel.

2.2 colored plane and color function

A „colored plane“ (P, f) is a (finite) set

$$P_{(m,n)} = \{(x, z) \in \mathbb{N} \times \mathbb{N} \;,\; 0 \leq x \leq m \;,\; 0 \leq z \leq n \;\}$$

and a function

$$f \,:\, P_{(m,n)} \;\longrightarrow\; \{0, 1\}$$

which we will call „color-function“.

(we will use P instead of $P_{(m,n)}$.)

2.3 spot of color 1

Let $S \subseteq \{(x,z) \in (P,f)\,,\, f(x,z) = 1\}$ **be a subset of color 1 in a colored plane** (P,f) **and**

$$z_{min} := \min_{(x,z)\in S} \{z\} \quad \textbf{and} \quad z_{max} := \max_{(x,z)\in S} \{z\}$$

S **is called a „spot of color 1“, if the following conditions are valid:**

- $\boxed{\textbf{for each row } z \textbf{ with } z_{min} < z \le z_{max}}$ **there exists** $\boxed{x_m(z) \in \mathbb{N}}$ **with** $(x_m(z), z) \in S$ **and** $(x_m(z), z-1) \in S$
- $\boxed{\textbf{for each row } z \textbf{ with } z_{min} \le z \le z_{max}}$ **there exist** $\boxed{x_a(z)}$ **and** $\boxed{x_e(z)}$ **such that** $x_a(z) \le x \le x_e(z) \quad \forall\,(x,z) \in S$

spot S of pixels of color 1:

z \ x	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0
5	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0
6	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0
7	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	0
8	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

picture 2.3:
emple of a „spot of color 1“

or if the following conditions are valid:

There exist

$$x_{min}(S) := \min_{z_{min} \le z \le z_{max}} \{x_{min}(z)\} \quad \textbf{and} \quad x_{max}(S) := \max_{z_{min} \le z \le z_{max}} \{x_{max}(z)\}$$

with

- $\boxed{\text{for each column } x \text{ with } x_{min}(S) < x \le x_{max}(S)}$
  there exists $\boxed{z_m(x) \in \mathbb{N}}$
  with $(x, z_m(x)) \in S$
  and $(x-1, z_m(x)) \in S$
- $\boxed{\text{for each column } x \text{ with } x_{min}(S) \le x \le x_{max}(S)}$
  there exist $\boxed{z_a(x)}$ **and** $\boxed{z_e(x)}$
  such that
  $z_a(x) \le z \le z_e(x) \quad \forall\, (x, z) \in S$

spot S of pixels of color 1:

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0
3	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0
4	0	0	0	1	0	1	1	1	1	0	0	0	0	0	0	0
5	0	0	0	1	0	1	1	1	1	0	0	0	0	0	0	0
6	0	0	0	1	0	1	1	1	1	0	0	0	0	0	0	0
7	0	0	0	1	0	1	1	1	0	0	0	0	0	0	0	0
8	0	0	0	1	0	1	1	1	0	0	0	0	0	0	0	0
9	0	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0
10	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0
11	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

picture 2.3a

2.4 area $a(S)$ of a spot S of color 1

For a spot S of color 1
- with rows between z_{min} and z_{max} and
- with functions $x_a(z)$ and $x_e(z)$ for each row z with $z_{min} \leq z \leq z_{max}$

we define the „area $a(S)$ of a spot S“

$$a(S) := \sum_{z=z_{min}}^{z=z_{max}} (x_e(z) - x_a(z) + 1)$$

or

$$a(S) := \sum_{x=x_{min}}^{x=x_{max}} (z_e(x) - z_a(x) + 1)$$

remark: In *picture 2.3* we have $a(S) = 9 + 10 + 9 + 8 + 7 + 5 = 48$.

2.5 interval $[P_1, P_2]$ in a spot S of color 1

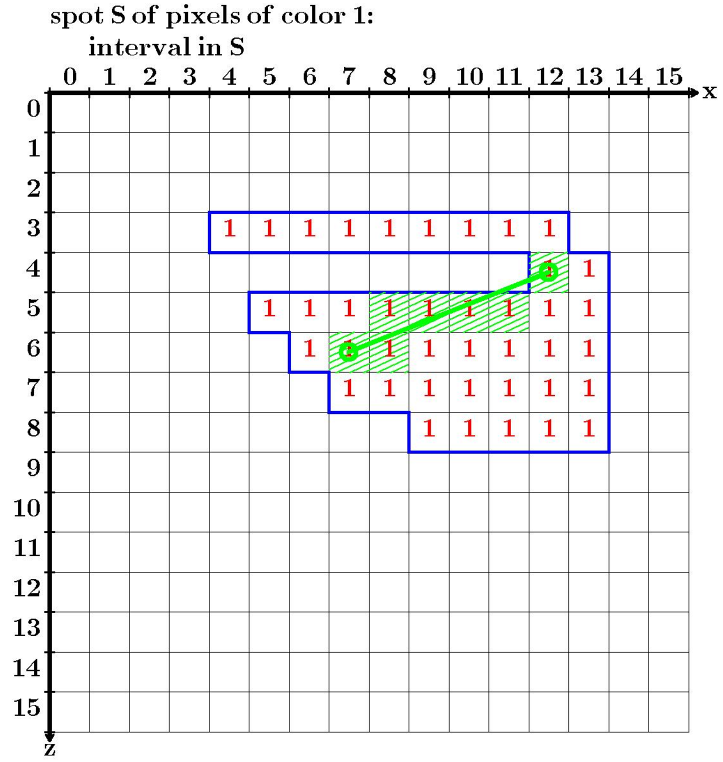
For two pixels

$P_1 = (x_1, z_1) \in S$ **and** $P_2 = (x_2, z_2) \in S$

the „interval $[P_1, P_2]$"

is defined as the set

$$I := \left\{ \begin{array}{l} (x, z) \in S\,, \\ \quad \exists\, u \;\; \textbf{with}\; x - 0.5 \le u < x + 0.5 \\ \quad \textbf{such that} \\ \quad z - 0.5 \le \\ \qquad \dfrac{(z_2 - z_1) \cdot u + (z_1 \cdot x_2 - x_1 \cdot z_2)}{x_2 - x_1} \\ \qquad\qquad\qquad\qquad < z + 0.5 \end{array} \right\}$$



picture 2.4

2.6 circumference U_S of a spot S of color 1

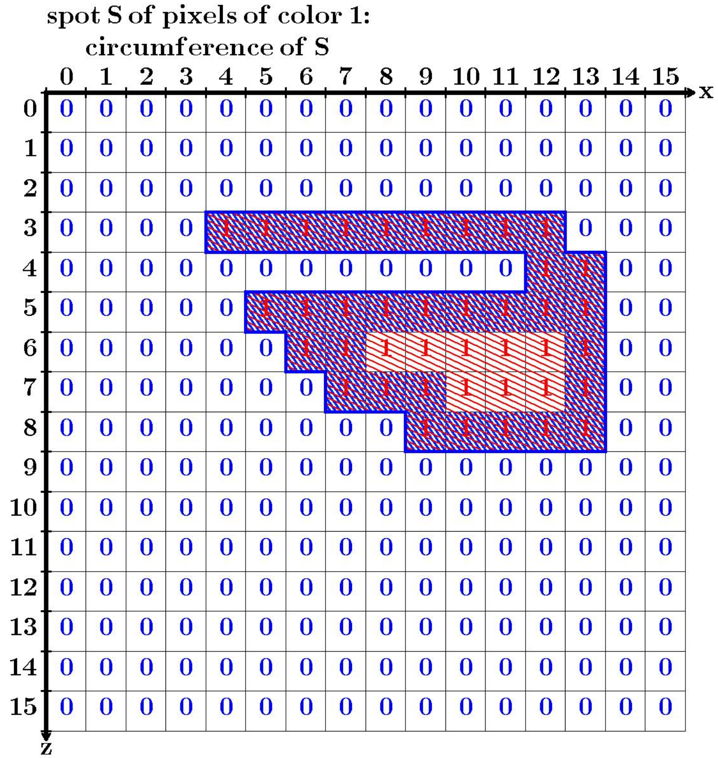
Let S be a spot of color 1 with rows between z_{min} and z_{max},

and let $x_a(z)$ and $x_e(z)$ be functions for each row z with $z_{min} \le z \le z_{max}$.

The ordered set

$$U_S := \left\{ \begin{matrix} \begin{pmatrix} x_a(z_{min}) \\ z_{min} \end{pmatrix}, & \begin{pmatrix} x_e(z_{min}) \\ z_{min} \end{pmatrix}, \\ \begin{pmatrix} x_a(z_{min}+1) \\ z_{min}+1 \end{pmatrix}, & \begin{pmatrix} x_e(z_{min}+1) \\ z_{min}+1 \end{pmatrix}, \\ \begin{pmatrix} x_a(z_{min}+2) \\ z_{min}+2 \end{pmatrix}, & \begin{pmatrix} x_e(z_{min}+2) \\ z_{min}+2 \end{pmatrix}, \\ \ldots, & \ldots, \\ \begin{pmatrix} x_a(z_{max}) \\ z_{max} \end{pmatrix}, & \begin{pmatrix} x_e(z_{max}) \\ z_{max} \end{pmatrix} \end{matrix} \right\}$$

of pixels of S is called „circumference U_S of the spot S of color 1“



picture 2.5:
circumference of S

Let S be a spot of color 1 with columns between x_{min} and x_{max},

and let $z_a(x)$ and $z_e(x)$ be functions for each column x with $x_{min} \leq x \leq x_{max}$.

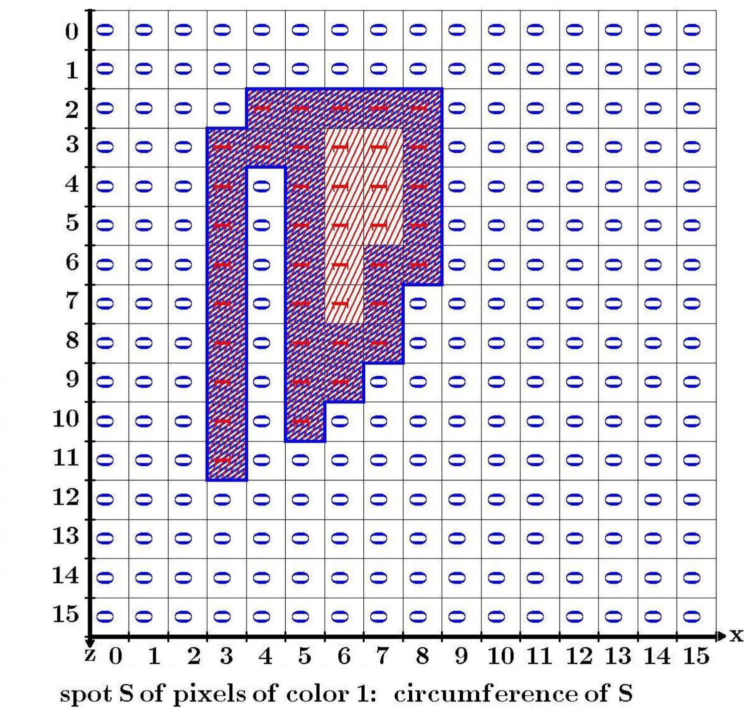
The ordered set

$$U_S := \left\{ \begin{array}{ll} \begin{pmatrix} z_a(x_{min}) \\ x_{min} \end{pmatrix}, & \begin{pmatrix} z_e(z_{min}) \\ x_{min} \end{pmatrix}, \\ \begin{pmatrix} z_a(x_{min}+1) \\ x_{min}+1 \end{pmatrix}, & \begin{pmatrix} z_e(z_{min}+1) \\ x_{min}+1 \end{pmatrix}, \\ \begin{pmatrix} z_a(x_{min}+2) \\ x_{min}+2 \end{pmatrix}, & \begin{pmatrix} z_e(z_{min}+2) \\ x_{min}+2 \end{pmatrix}, \\ \ldots, & \ldots, \\ \begin{pmatrix} z_a(x_{max}) \\ x_{max} \end{pmatrix}, & \begin{pmatrix} z_e(x_{max}) \\ x_{max} \end{pmatrix} \end{array} \right\}$$

or

of pixels of S is called

„circumference U_S of the spot S of color 1“



picture 2.5a):
circumference of S

remark 2.5a)

Let $x_a(S) := \min\limits_{z_{min} \le z \le z_{max}} \{x_a(z) \in S\}$ **and** $x_e(S) := \max\limits_{z_{min} \le z \le z_{max}} \{x_e(z) \in S\}$**.**

$\boxed{\textbf{For each } x \textbf{ with } x_a(S) \le x \le x_e(S)}$ **there exist pixels** $\boxed{(x, z_P) \in U_S}$**.**

remark 2.5b)

Let $x_a(S) := \min\limits_{z_{min} \le z \le z_{max}} \{x_a(z) \in S\}$
and $x_e(S) := \max\limits_{z_{min} \le z \le z_{max}} \{x_e(z) \in S\}$**.**

$\boxed{\textbf{For each } x \textbf{ with } x_a(S) \le x \le x_e(S)}$ **there exist pixels** $\boxed{(x_P, z) \in U_S}$

2.7 length $d(U_S)$ of the circumference of a spot S of color 1

The length $d(U_P)$ of the circumference of a pixel P is defined as 4.

The length $d(U_S)$ of the circumference of a spot S of color 1
with rows between z_{min} and z_{max}
and $x_a(z) \leq x \leq x_e(z)$ for each row z with $z_{min} \leq z \leq z_{max}$
is defined as

$$\begin{aligned} d(U_S) :&= (x_e(z_{min}) - x_a(z_{min}) + 1) \\ &+ \sum_{z=z_{min}}^{z=z_{max}-1} [2 + |x_a(z) - x_a(z+1)| + |x_e(z) - x_e(z+1)|] \\ &+ (x_e(z_{max}) - x_a(z_{max}) + 1) \end{aligned}$$

remark: In *picture 2.5* we have

$$\begin{aligned} d(U_S) &= (12 - 4 + 1) \\ &\quad + [2+1+1] + [2+1+0] + [2+1+0] + [2+1+0] + [2+2+0] + (13-9+1) \\ &= 9 + 4 + 3 + 3 + 3 + 4 + 5 = 31 \end{aligned}$$

2.8 dense spot

In *picture 2.1* we chose the coordinate-system in a „natural way“ - that means without any rotation. But if the cut of martensite steel is rotated, then it is possible, that a spot which has cavities of small dimension in one position can loose this property after rotation (we will show this in part 2.14).
Therefore we will define a

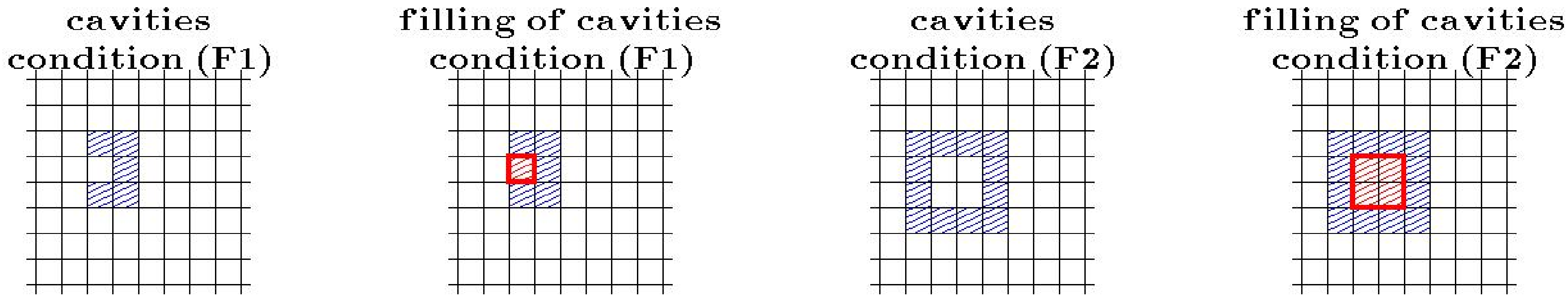
$$\boxed{\textbf{dense spot}}$$

if all cavities which are

$$\boxed{\textbf{smaller than } \gamma^2 \textbf{ with } \gamma = 0.5\,\mu\,m = \tfrac{1}{2} \cdot \textbf{width of a lance}}$$

are filled with color 1.

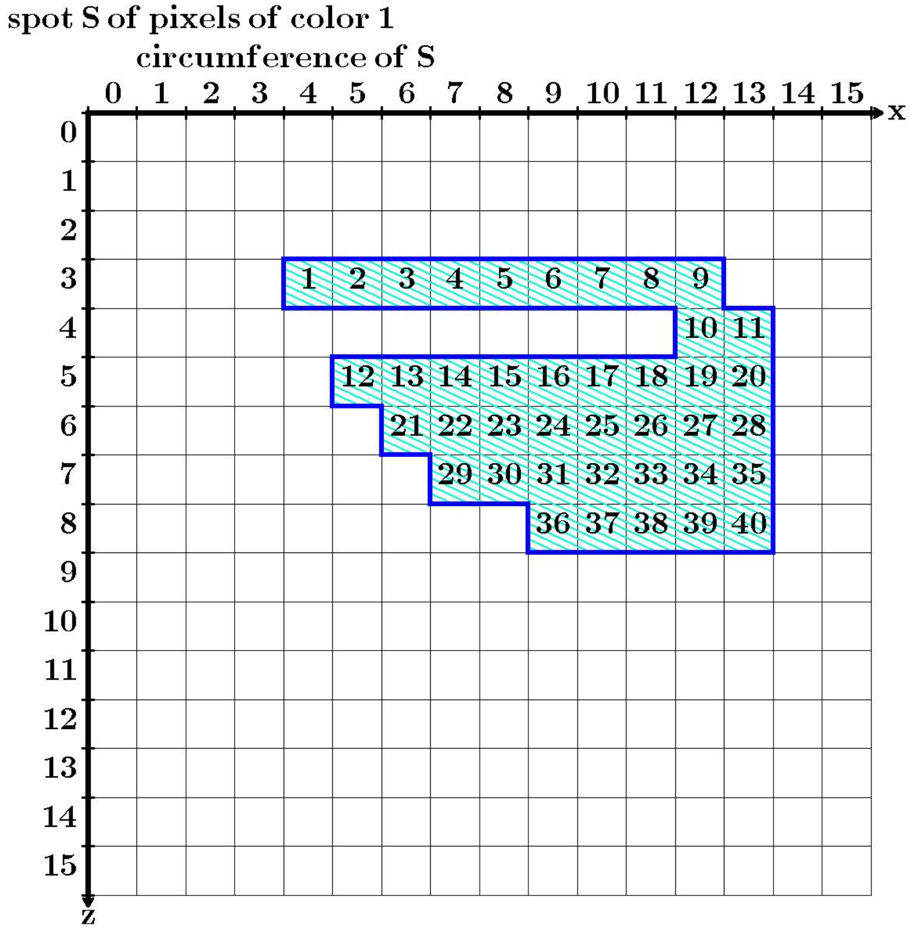
Therefore we fill all cavities which are smaller than γ^2.



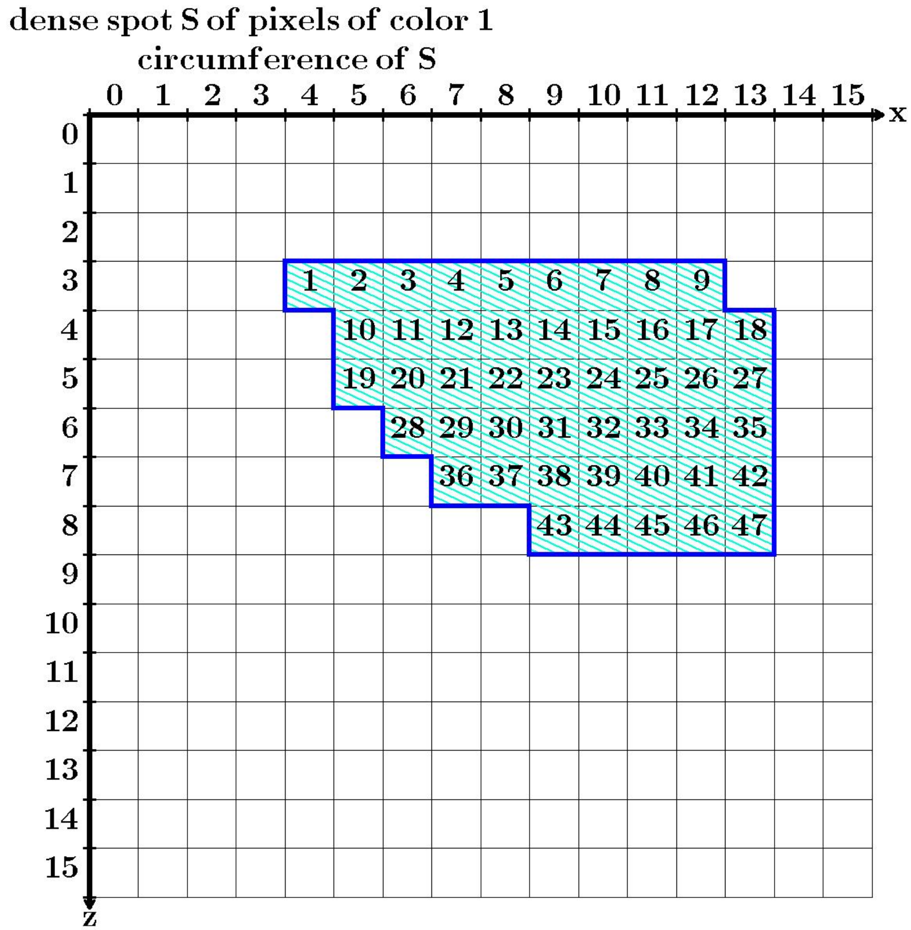
***picture 2.6*: cavities and its filling**

remark: $1\,\mu\,m =$ **width of 5 pixels** **or** $\gamma^2 \approx 2 \times 2$ **pixels.**

example *picture 2.3*: cavities and its filling for getting a dense spot

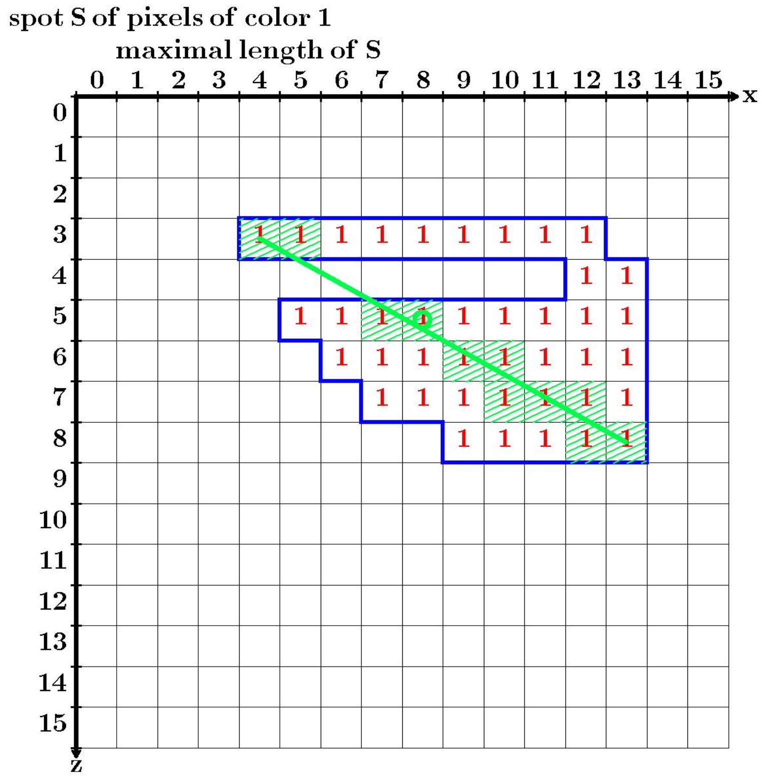


picture 2.3:
numbered elements of the spot S



picture 2.7:
spot S, filled with cavities, which are smaller than γ^2

2.9 distance $d(P_1, P_2)$ between two pixels P_1 and P_2



picture 2.8

The „distance between two pixels“
$P_1 = (x_1, z_1)$ **and** $P_2 = (x_2, z_2)$
is defined as the euclidean distance

$$d\,(P_1, P_2) = \sqrt{(x_1 - x_2)^2 + (z_1 - z_2)^2}.$$

2.10 maximal length h_S of a spot S

For a spot S of color 1 there exists a real number h_S with

$$h_S = \max_{P_1, P_2 \in U_S} \{d\,(P_1, P_2)\}$$

h_S **is called the „maximal length of S“.**

remark:
In em picture 2.8, we have $P_1 = (4, 3)$ and $P_2 = (13, 8)$
and therefore $\boxed{h_S = \sqrt{5^2 + 9^2} = \sqrt{106} \approx 10.3}$

remark 2:
In general, the maximal lenght can be represented not only by P_1 and P_2, but also by other pixels.

2.11 midpoint of an interval $[P_1, P_2]$ of a spot S

Let $P_1 = (x_1, z_1) \in S$ and $P_2 = (x_2, z_2) \in S$. We define a „midpoint“

$$M(P_1, P_2) = (x_M, z_M) := \left(INT\left[\frac{1}{2} \cdot (x_1 + x_2)\right], INT\left[\frac{1}{2} \cdot (z_1 + z_2)\right] \right)$$

2.12 width of a dense spot S

If $M(P_1, P_2)$ is not a pixel of the dense spot S, a „width“ of the spot S is not defined.

Now let us assume that S is a dense spot and $M(P_1, P_2)$ is a pixel of S.
We choose two pixels of S with

$$d(P_1, P_2 = h_S$$

and

$$M(P_1, P_2) \text{ is a pixel of } S$$

Then we construct a line $g_\perp$ with $M(P_1, P_2) \in g_\perp$, which is normal to the line $\overline{P_1, P_2}$:

$$g_\perp := \left\{ (x, z) \in \mathrm{I\!R}^2 , (z_1 - z_2) \cdot (z - z_M) = -(x_1 - x_2) \cdot (x - x_M) \right\}$$

This line meets the circumference U_S in (at least) two points which we call P_3 and P_4.
We choose

$$P_3 = (x_3, z_3) \in U_S \text{ and } P_4 = (x_4, z_4) \in U_S \text{ with } x_3 \le x_4$$

remark: If the midpoint $M(P_1, P_2)$ is a pixel of the dense spot S, then the two points P_3 and P_4 and also the distance $d(P_1,\, P_2)$ are unique determined.

The „width of the dense spot S“ is defined as

$$\boxed{w_S := \max_{d(P_1,P_2)=h_S} d(P_3, P_4), \text{ if } M(P_1, P_2) \text{ is a pixel of } S}$$

2.13 elliptic spot

elliptic spot $S(h_S,\, w_S)$

Let S be a dense spot.
For every quadrupel $[P_1,\, P_2,\, P_3,\, P_4]$ with $d(P_1, P_2) = h_S$, where $M(P_1, P_2)$ is a pixel of S, and where w_S is the width of S
we will say that S is an

„elliptic spot $S(h_S,\, w_S)$“

if

$$7 \cdot w_S \leq h_S$$

2.14 direction of an elliptic spot

In an elliptic spot S,

- **let n be the number of pairs $(P_1,\, P_2)$ where $P_1,\, P_2 \in U_S$ with $d(P_1, P_2) = h_S$, and**
- **for $i \le n$ let α_i be the direction of $\overline{P_{1,i},\, P_{2,i}}$, and**
- **let $w_{S,i} := \max\limits_{d(P_{1,i}, P_{2,i})=h_S} d(P_{3,i},\, P_{4,i})$ with $h_S \ge 7 \cdot w_{S,i}$.**

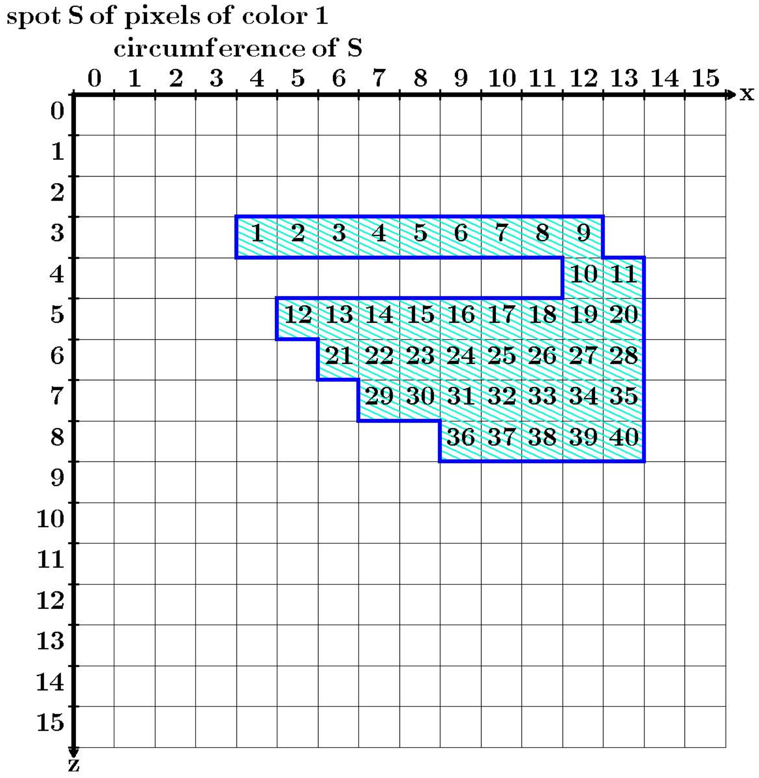
Then the „direction α of the elliptic spot“ is defined as

$$\boxed{\alpha = \frac{1}{n} \cdot \sum_{i=1}^{n} \alpha_i}$$

2.15 (appendix) pixel-rotation

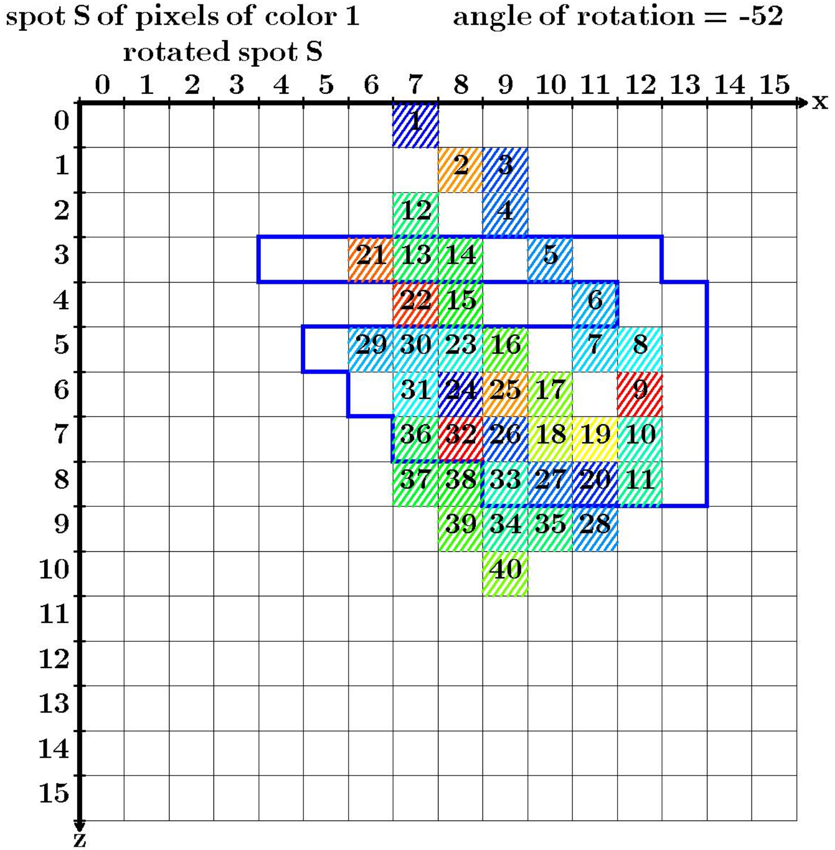
We define the „center (x_C, y_C) of a spot $S = \left\{(x_i, z_i) \in \mathbb{N}^2\,,\, 1 \le i \le n\right\}$“ in the natural way:

$$x_C := INT\left(\frac{1}{n}\cdot\sum_{i_1}^{n} x_i\right) \quad \textbf{and} \quad y_C := INT\left(\frac{1}{n}\cdot\sum_{i_1}^{n} y_i\right)$$

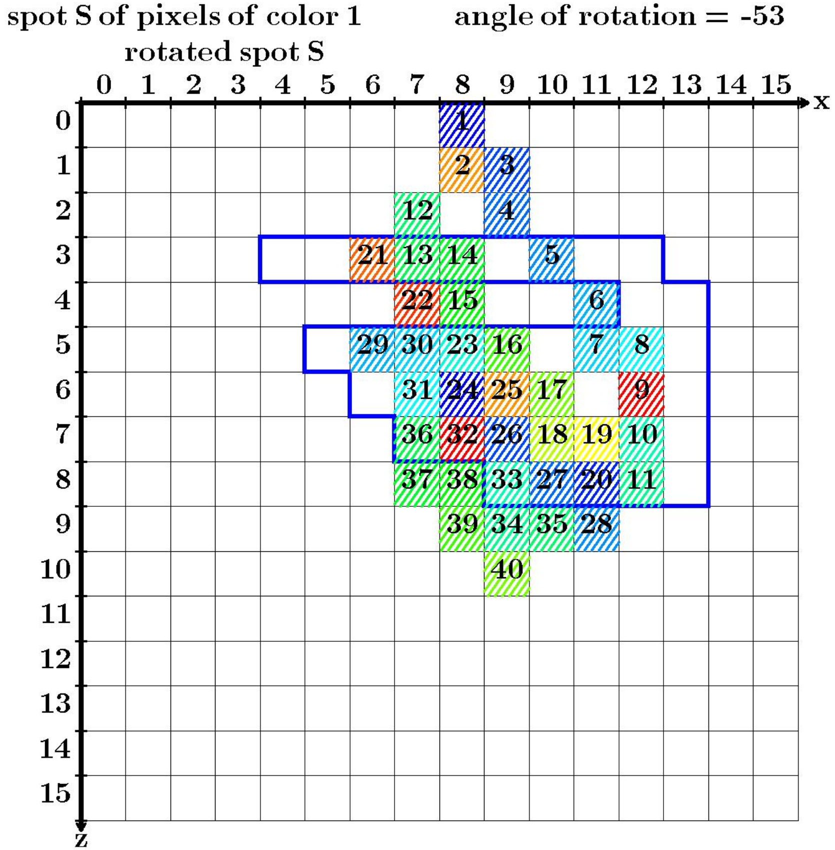


picture 2.9: **numbered elements of *picture 2.3***

Let us rotate all pixels of the spot S:



picture 2.10:
spot S rotated at angle -52°
with the center of S



picture 2.11:
spot S rotated at angle -53°
with the center of S

result: it is necessary to fill all cavities after rotation (and also before).

Therefore we introduced „dense spots“!

3 statistical solutions

3.1 Hough-Transformation

Again we want to determine the direction of one lance in martensite steel.

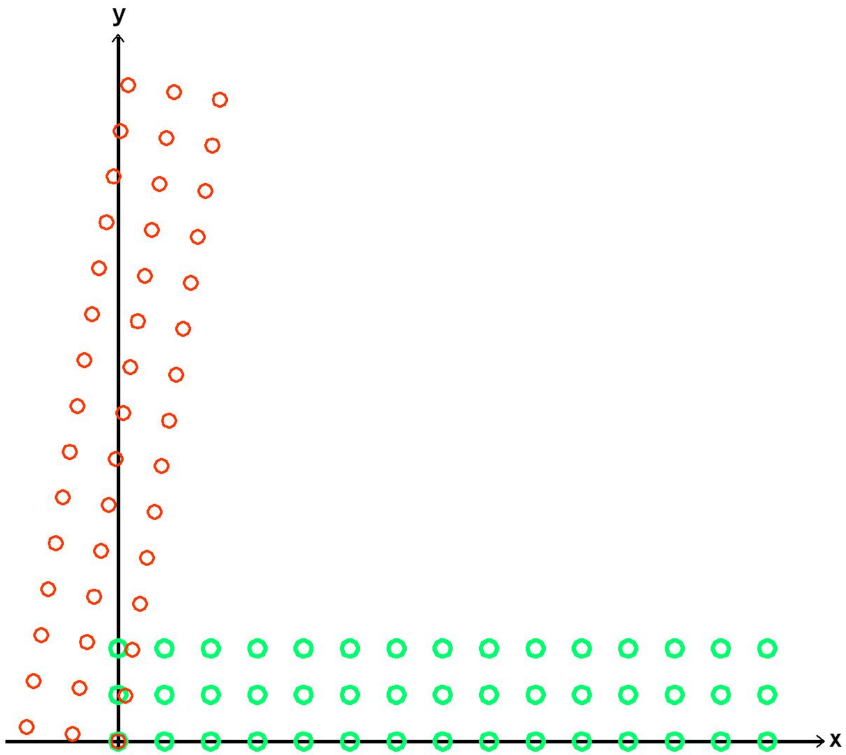
That means:

For a grid of pixels of color 1 we want to define a direction

We consider a grid of $m \times n$ pixels.
let t be the number of pairs (P_1, P_2) of pixels of color 1, with $P_1 \neq P_2$.

1. **For every pair (P_1, P_2) we calculate the direction α of $\overline{P_1, P_2}$**
2. **for every angle α $(0 \leq \alpha < 180^\circ)$ we calculate the probability $W(\alpha)$ for the direction α.**

3.2 example



picture 3.1:

green spot with main direction $\alpha = 0^\circ$ and maximal direction $\varphi_{max} = \pm 8^\circ$.
red spot with main direction $\alpha = 0^\circ$ and maximal direction $\varphi_{max} = 81^\circ \pm 9^\circ$.

If we calculate

the probability $W(\alpha)$ for every angle α ($0 \leq \alpha < 180°$)

then we will find:

green spot

there are 990 direction vectors α_i,
and it is $-1.5° \leq \alpha_i \leq 1.5°$ for 32 % of all these direction vectors
(this result is in accordance with our expectation)

red spot

there are 990 direction vectors α_i,
and it is $73.5° \leq \alpha_i \leq 76.5°$ for 16 % of all these direction vectors
and it is $76.5° < \alpha_i \leq 79.°$ for 14 % of all these direction vectors
(this result is not in accordance with our expectation)

3.3 regression

There exists the procedure „simple linear regression“, and it is possible to use this procedure to find a direction of a spot. But there are two „lines of regression“, and no one of these lines are in accordance with our expectation.

$$
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\end{matrix}
$$

Thank you for listening