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Nano-point-geometry
for use in material science

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Nano-point-geometry for use in material science

1 microstructure

2 determination of height of voxels

3 landscape of voxels

4 roughening of a surface (Sandstrahlen)

0. Introduction

There exist different kinds of surfaces - two of these are

- **surfaces in the differential geometry (line-geometry)**
- **surfaces of real objects (point-geometry)**

Especially the characterisation of the surface

of a planar metallic plate

during different steps of a plastic deformation

needs a geometric description of point-geometry, because microscopy gives point-information in a coordinate-system of micro- or nano-meters.

When we examine the microstructure of a material we are looking at a very small sample of a stick (e.g. $1 \times 2\mu\, m^2$, produced by a microscope, recorded as $1\,0000 \times 2\,000$ pixels with area $1 \times 1\, nm$) of a quite pure material.

From this limited view we try to understand how the properties of the whole stick ($20 \times 100mm^2 = 2 \cdot 10^4 \times 10^5 \mu\, m^2$) relate to its small sample - if there is a relation (remember the cracks in the wheels of trains).

But, when we measure only the macroscopic properties of material (such as tensile strength[1]), hardness, density or electrical conductivity), we use a much larger specimen - in the majority of cases these specimens are a mean of all small samples.

It should <u>not</u> be surprising, therefore, that it is difficult thus to establish correlations between macroscopic properties and the microstructure in a „representative“ sample.

[1]) tensile strength= Zugprobe

1 Microstructure

1.1 Aluminium, basic principles

atomic radius: **0.143 *nm***

cells of (8+6=14) aluminium-atoms in a face centered cube (fcc):
lattice constant: **a = 0.404 *nm***

lattice of (monocrystal) $\alpha-$aluminium which is constructed by these cells

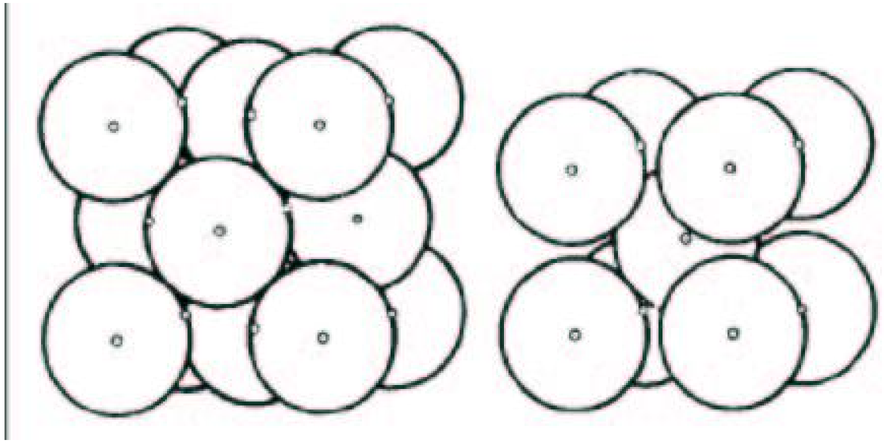


fig. 1.1.1:
left: fcc - right: bcc
kubisch - kubisch
flächenzentriert - raumzentriert

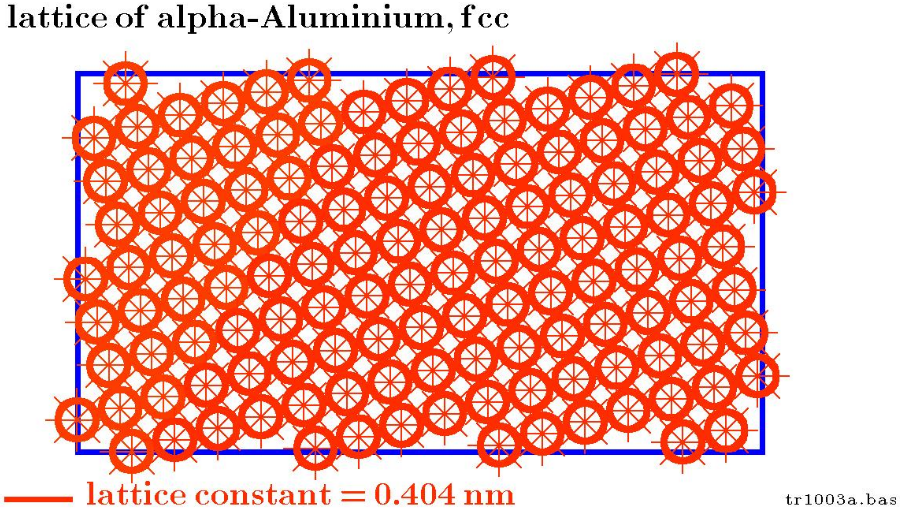


fig. 1.1.2:

grid of α-aluminium (monocrystal)

lattice vacancy and dislocation (that means a non-occupied place in a regular lattice) is a crystallographic defect, or irregularity, within a crystal structure of the atoms. Theoretically only at temperature of 0 K all regular lattice sites are occupied. But especially during crystallisation from a liquid to a solid phase we usually find

$$10^8 \textbf{ lattice vacancies per } cm^3 \textbf{ at } 20^\circ C,$$

and the concentration of lattice vacancies increases exponentally with temperature.

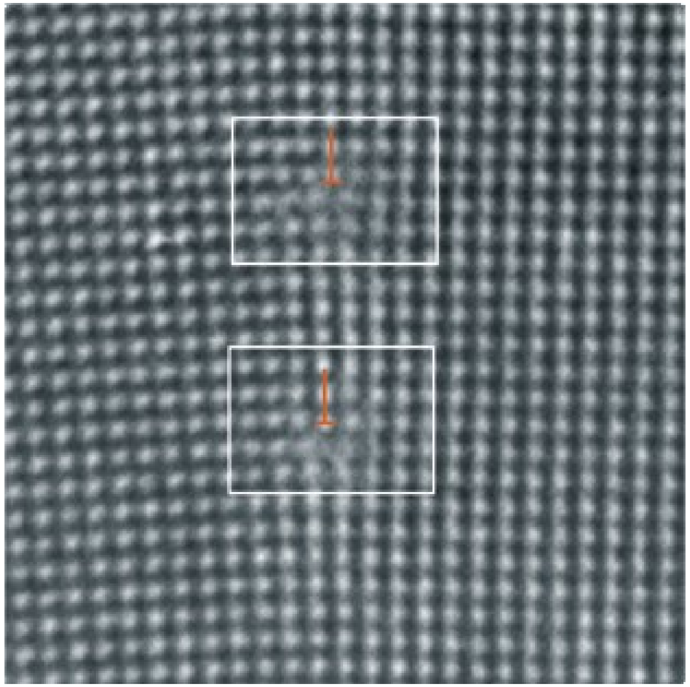


fig. 1.1.3:
lattice vacancy and dislocation
10^8 **lattice vacancies per** cm^3 **at 20**$^\circ C$

$(6 \times 6\, nm^2)$

The presence of dislocations strongly influences many of the properties of materials.

Therefore in nano-geometry we have no continuity and no lines - but points and statistics.

1.2 Macrostructure: Plastic deformation of metals

When a crystal is deformed elastically under influence of applied stresses, it returns to its original state upon removal of the stresses.

However, if the applied stresses are sufficiently large, a certain amount of deformation remains after removal of the stresses: the crystal has been plastically deformed.

Experimental facts for monocrystal Aluminia (Einkristalle aus Aluminium):
We assume that we have a monocrystal (99.5% Al). Then we have a (nearly) regular lattice of atoms (*fig. 1.1.2*) shows a 2-dimensional cut). But each of these atoms (at „normal“ temperatures) is always busy.

We want to understand (or to model) the process of cracking - which starts with elastic deformation, and with increase of tension begins a plastic deformation and ends with a crack.

Here we study the dependence between tension and the appearance of „slip bands“, which result from the sliding of one part of a crystal relative to another, and we want to determine the height and the slip-direction of each slip band in dependence of tension.
Plastic deformation is inhomogenous in the sense that that only a small number of atoms actually take part in the slip process.

„Real“ materials are not monocrystal: during solidification cluster or „dendrits“ are produced from cells of atoms, and after solidification we find „grains“, which have uncalculable structures - and in this case slip bands occure at the boundaries of grains.

1. **all microstructures are irregular in the sense that no two grains in a specimen of metal are of exactly the same size and shape,**
2. **nevertheless all this different grains of the structure are fitted together to fill space (therefore it is not possible that they all are convex),**

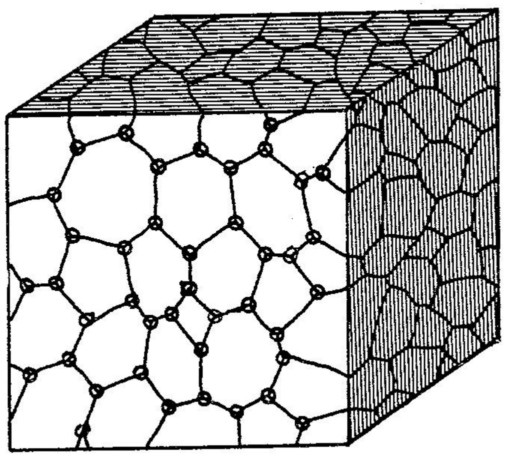


fig. 1.2.1: **grains**
$(50 \times 50 \times 50\, \mu m^3)$

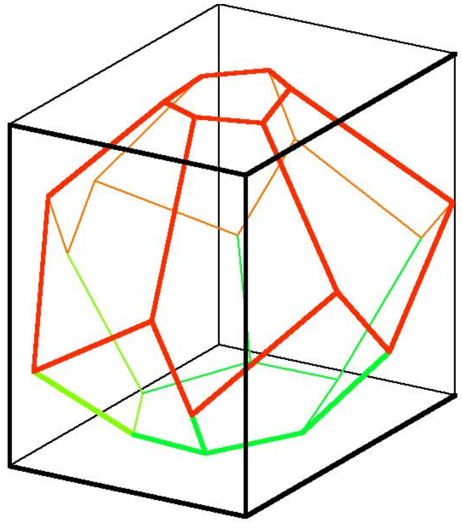


fig. 1.2.2:: **grain, 3D**
$(15 \times 15 \times 15\, \mu m^3)$, $2 \cdot 10^{14} atoms$

Therefore we use monocrystal materials for our research.

slip process due to a tensile stress (=Dehnungs-Beanspruchung)

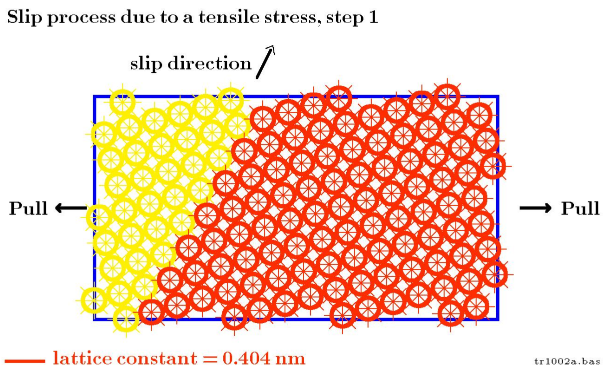


fig. 1.2.3:

the right part starts slipping

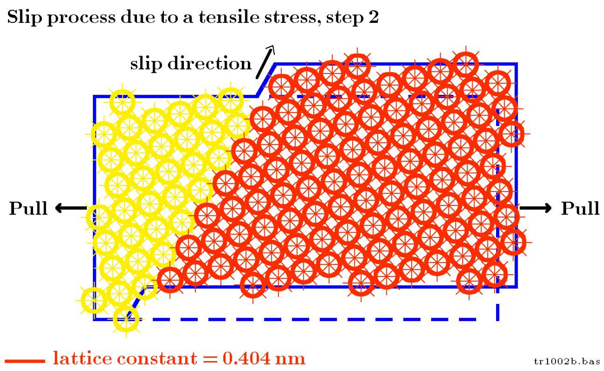


fig. 1.2.4:

the right part starts slipping

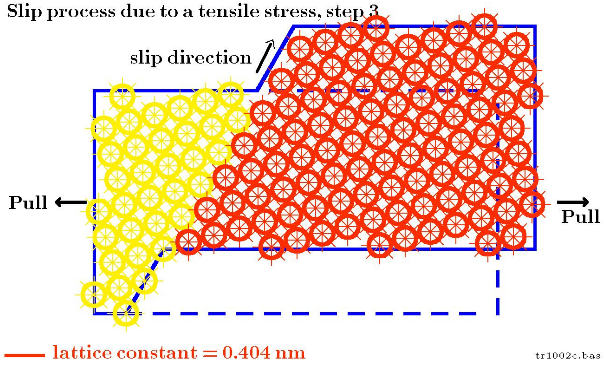


fig. 1.2.5:

the right part slips

It may be that this slipping reach a height of 10 nm - or 200 slip steps.

(a human hair has a diameter of about 70 nm).

But we have to remember, that our drawing neglects:

10^8 **lattice vacancies per** cm^3 **at 20**$^\circ C$

In a 3D-sample the result of a slipping process is called „slip band“.

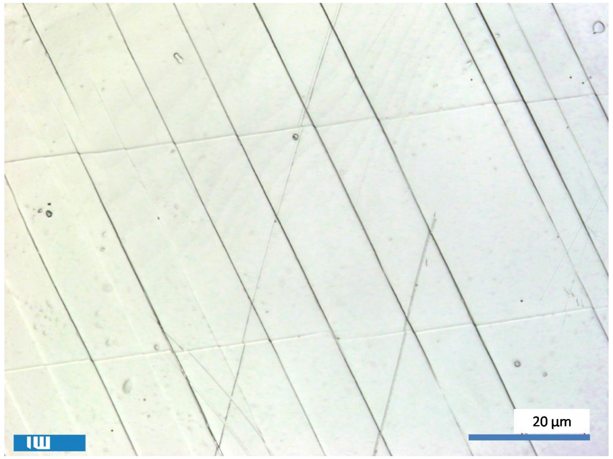


fig. 1.2.6:
Al, $100 \times 70\,\mu m$
$\varphi_Z = -0.05$, **95%**

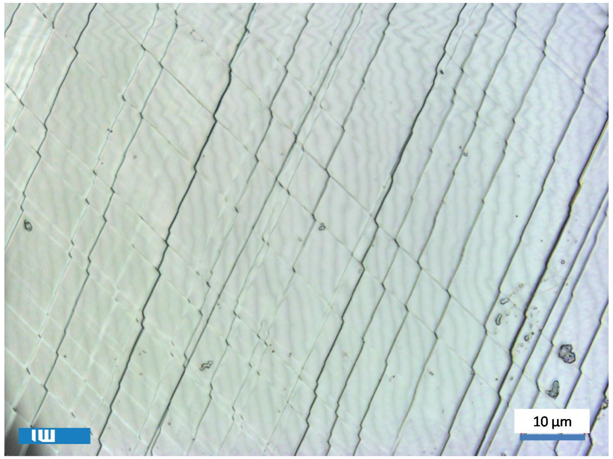


fig. 1.2.7:
Al, $88 \times 62\,\mu m$
$\varphi_Z = -0.10$, **90%**

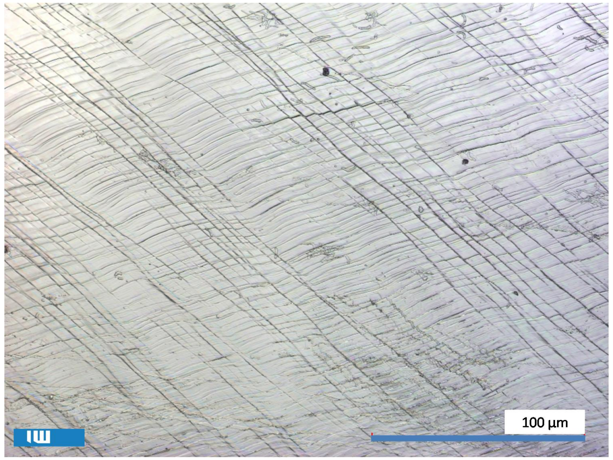


fig. 1.2.8:
Al, $285 \times 200\,\mu m$
$\varphi_Z = -0.16$, **85%**

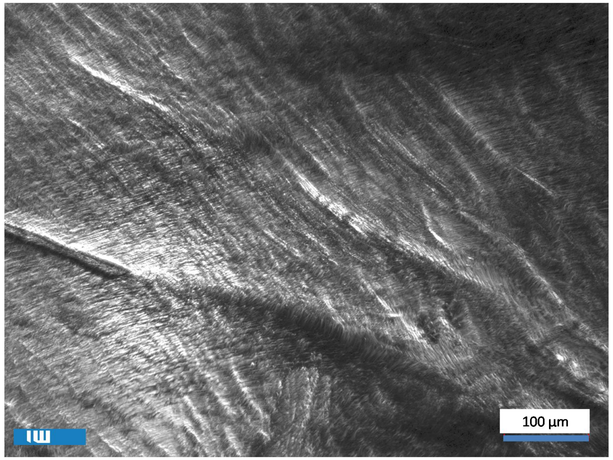


fig. 1.2.9:
Al, $800 \times 560\,\mu m$
$\varphi_Z = -0.43$, **65%**

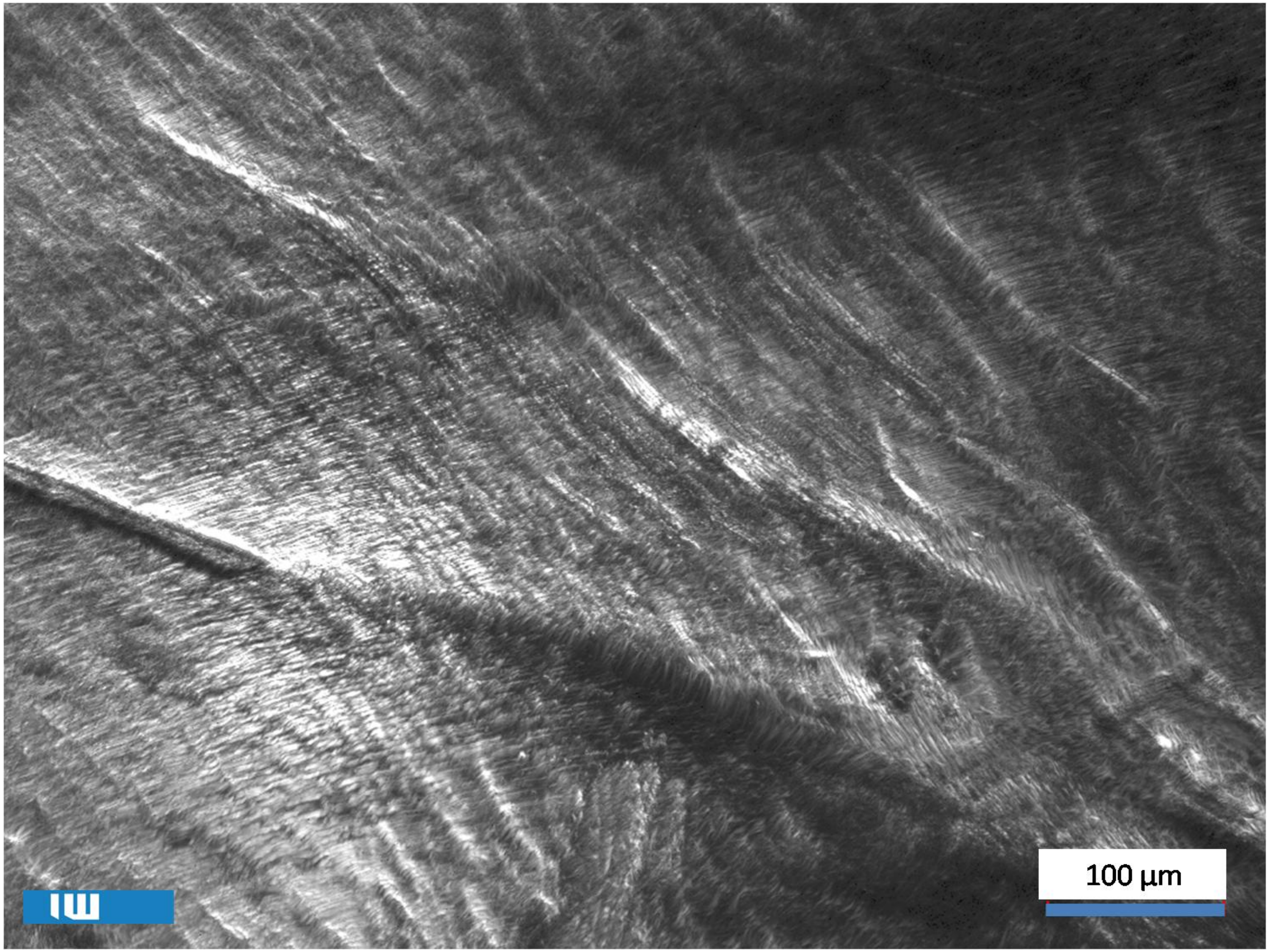


fig. 1.2.9: **slip bands**

2 determination of height of voxels

We determine the surface of our stick of α-aluminium by a special laser-microscope. The result of this procedure is a grid of $n \times m$ pixels where each pixel has its own height: we will call it „voxel“.

From a very small part $\mathcal{P}$ of the surface of the stick we get pixels $p(i, j)$ with $-n_i \leq i \leq n_i$ und $-n_j \leq j \leq n_j$ and also the distances $d(i, j)$ between the microscope and the pixel $p(i, j)$. From this we have to determine th height $z(i, j)$ of the voxel $V(i, j, z(i, j))$.

In a first step we assume that the microscope has a position „perpendicular“ to $\mathcal{P}$.

What is „perpendicular“ to $\mathcal{P}$?

We introduce a simple model:

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
		9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9
1	9	48	48	48	48	41	41	41	41	41	34	34	34	34	34	27	27	27	27	27
2	8	48	48	48	48	48	41	41	41	41	41	34	34	34	34	34	27	27	27	27
3	7	55	48	48	48	48	48	41	41	41	41	41	34	34	34	34	34	27	27	27
4	6	55	55	48	48	48	48	48	41	41	41	41	41	34	34	34	34	34	27	27
5	5	55	55	55	48	48	48	48	48	41	41	41	41	41	34	34	34	34	34	27
6	4	55	55	55	55	48	48	48	48	48	41	41	41	41	41	34	34	34	34	34
7	3	55	55	55	55	55	48	48	48	48	48	41	41	41	41	41	34	34	34	34
8	2	62	55	55	55	55	55	48	48	48	48	48	41	41	41	41	41	34	34	34
9	1	62	62	55	55	55	55	55	48	48	48	48	48	41	41	41	41	41	34	34
10	0	62	62	62	55	55	55	55	55	48	48	48	48	48	41	41	41	41	41	34
11	-1	62	62	62	62	55	55	55	55	55	48	48	48	48	48	41	41	41	41	41
12	-2	62	62	62	62	62	55	55	55	55	55	48	48	48	48	48	41	41	41	41
13	-3	69	62	62	62	62	62	55	55	55	55	55	48	48	48	48	48	41	41	41
14	-4	69	69	62	62	62	55	62	55	55	55	55	55	48	48	48	48	48	41	41
15	-5	69	69	69	62	62	62	62	62	55	55	55	55	55	48	48	48	48	48	41
16	-6	69	69	69	69	62	62	62	62	62	55	55	55	55	55	48	48	48	48	48
17	-7	69	69	69	69	69	62	62	62	62	62	55	55	55	55	55	48	48	48	48
18	-8	69	69	69	69	69	69	62	62	62	62	62	55	55	55	55	55	48	48	48
19	-9	69	69	69	69	69	69	69	62	62	62	62	62	55	55	55	55	55	48	48

table 2.0.1: $19 \cdot 19 = 361$ **voxels**

(ground area 1×1 **units) in a plane, then 7 units upstairs**

2.1 perpendicularity

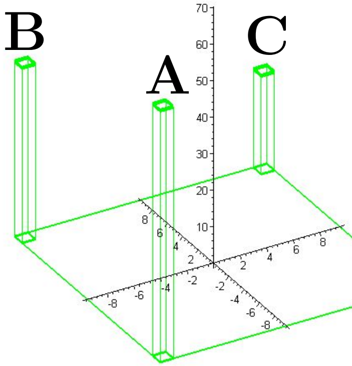


fig. 2.1.1: **position of the four vertices with levels** $A = (-9, -9, 69)$**,** $B = (-9, 9, 48)$

$C = (9, 9, 27)$**,** $D = (9, -9, 48)$

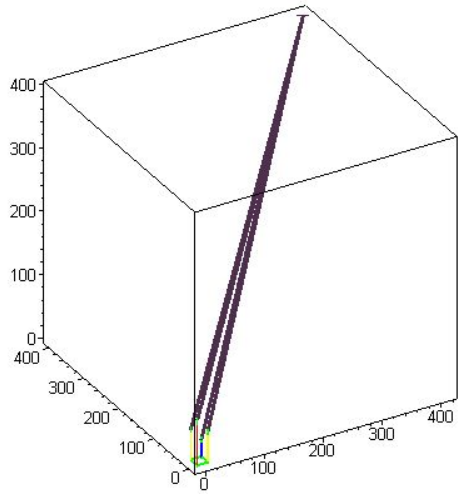


fig. 2.1.2: **position of a microscope which is used to determine the height of voxels:**

$P = (438, 438, 424)$

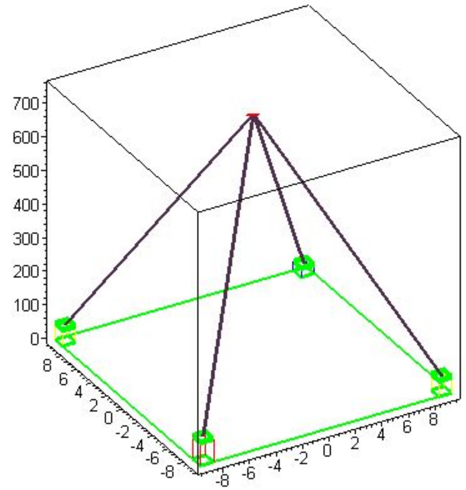


fig. 2.1.3: **position of a microscope which is used to determine the height of voxels:**

$P_0 = (0, 0, 751)$

In *fig. 2.1.2* we use a point P such that the distances between P and the 4 voxels are equal ($d(P, A) = d(P, A) = d(P, A) = d(P, A) = 725$).

In *fig. 2.1.3* we use a point P_0 such that P_0 is a point of the $z-$axis and the distance between $(0, 0, 0)$ and P_0 is equal to the distance between $(0, 0, 0)$ and P.

If we use the microscope-position P_0 then we will get the results of *table 2.0.1*, which looks like *fig. 2.1.4.* - But if we use the microscope-position P then we will get another result:
fig. 2.1.5 **shows, that the four vertices have the same height. And *fig. 2.1.6* shows that it seems that there is no slip band between the 361 voxels, because the height differs between 45 and 52 (and not between 27 and 69)**

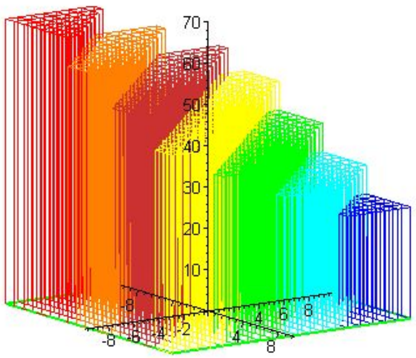


fig. 2.1.4:
voxels of *table 2.0.1*

position of the microscope:

$P_0 = (0, 0, 751)$

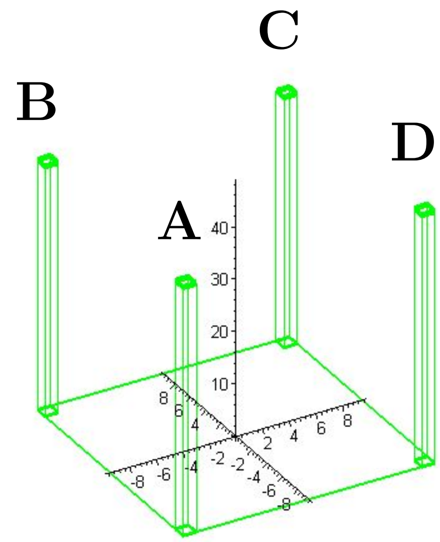


fig. 2.1.5:
position of the four vertices
position of the microscope:
$P = (438, 438, 424)$
with levels
$A = (-9, -9, 48)$, $B = (-9, 9, 48)$

$C = (9, 9, 48)$, $D = (9, -9, 48)$

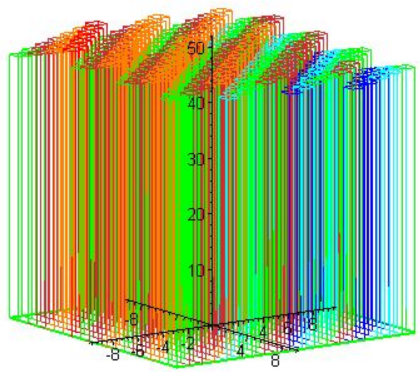


fig. 2.1.6:
voxels of *table 2.0.1*

position of the microscope:

$P = (438, 438, 424)$

	9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9
9	47	48	49	49	46	47	47	48	49	46	46	47	47	48	45	45	46	46	47
8	47	48	48	49	49	46	47	48	48	49	46	46	47	47	48	45	45	46	46
7	50	47	48	48	49	49	46	47	48	48	49	46	46	47	47	48	45	45	46
6	49	50	47	48	48	49	50	46	47	48	48	49	46	46	47	47	48	45	45
5	49	49	50	47	48	48	49	50	47	47	48	48	49	46	46	47	47	48	45
4	48	49	50	50	47	48	48	49	50	47	47	48	48	49	46	46	47	47	48
3	48	48	49	50	50	47	48	48	49	50	47	47	48	48	49	46	46	47	47
2	51	48	48	49	50	50	47	48	49	49	50	47	47	48	48	49	46	46	47
1	50	51	48	49	49	50	50	47	48	49	49	50	47	47	48	48	49	46	46
0	50	50	51	48	49	49	50	50	47	48	49	49	50	47	47	48	48	49	46
-1	49	50	50	51	48	49	49	50	50	47	48	49	49	50	47	47	48	48	49
-2	48	49	50	50	51	48	49	49	50	50	47	48	49	49	50	47	47	48	48
-3	51	48	49	50	50	51	48	49	49	50	50	47	48	49	49	50	47	47	48
-4	51	51	49	49	50	50	51	48	49	49	50	50	47	48	49	49	50	47	47
-5	50	51	51	49	49	50	50	51	48	49	49	50	50	47	48	49	49	50	47
-6	50	50	51	51	49	49	50	50	51	48	49	49	50	50	47	48	48	49	50
-7	49	50	50	51	51	49	49	50	50	51	48	49	49	50	50	47	48	48	49
-8	48	49	50	50	51	51	49	49	50	50	51	48	49	49	50	50	47	48	48
-9	48	48	49	50	50	51	51	49	49	50	50	51	48	49	49	50	50	47	48

table 2.1.1: **determination of the height of *table 2.0.1* with a microscope-position** $P = (438, 438, 424)$

$19 \cdot 19 = 361$ **levels between 45 and 52**

There is no possibility for determinating a slip band in this view of $\mathcal{P}$ of the surface of the stick. And we did not find a perpendicular to $\mathcal{P}$. - We need new concepts.

3 landscape of voxels

3.1 landscape with hills and valleys

Let us start with a simple example: We construct a „landscape“with hills and valleys, and we will define an „orientation of a valley“ and the detection of slip bands, their height and their orientation.

6	6	5	5	4	4	3	2	0	0	1	2	1	0	0	2	4	6	8	9	9	8	7	5	3	1	0	0	1	2
5	5	4	4	3	3	2	1	0	1	1	2	1	0	0	1	5	7	9	8	7	6	5	3	2	0	1	1	1	2
4	4	4	3	4	3	2	1	0	1	1	2	2	1	0	3	6	9	9	8	7	4	2	0	0	1	2	2	2	3
4	4	4	5	5	4	3	2	0	1	1	2	1	0	2	4	7	9	7	5	3	1	0	0	1	2	3	3	3	4
3	4	5	6	6	5	4	3	0	2	2	1	1	0	2	4	7	9	7	4	2	0	0	1	1	2	3	4	4	5
4	5	6	6	7	7	6	6	5	0	1	2	1	0	3	5	7	9	8	6	4	2	0	1	1	2	3	4	5	6
5	6	8	8	8	8	7	8	8	6	0	1	1	0	2	5	8	9	8	7	5	3	0	1	2	3	4	5	6	7
7	8	9	9	9	9	9	9	9	9	5	0	1	1	0	4	6	9	7	6	4	2	1	0	1	2	3	4	5	6
9	9	8	8	8	9	9	8	8	9	9	4	0	0	0	3	7	9	8	6	5	3	2	0	0	1	2	3	4	5
8	8	7	6	6	7	8	8	7	8	9	9	4	2	0	4	7	9	9	7	6	5	3	1	0	1	2	3	4	5
7	6	5	5	5	6	7	7	7	8	8	9	9	5	2	0	5	7	9	8	7	5	4	2	1	0	1	2	3	3
5	4	2	3	4	3	6	5	6	5	7	8	9	9	6	2	0	2	9	9	8	8	7	5	3	1	0	1	2	2
3	1	0	1	2	1	4	4	5	4	6	7	9	9	6	3	0	0	3	9	8	7	6	5	3	2	1	0	0	0
3	2	1	0	0	1	2	2	3	4	5	7	9	9	7	4	2	0	1	4	9	7	6	6	5	4	3	1	1	1
2	2	1	0	0	0	0	2	3	3	5	7	9	9	7	4	3	2	0	5	9	9	8	7	6	5	4	3	2	2
3	3	2	1	1	1	1	0	1	2	4	6	8	9	9	8	6	4	2	0	4	9	9	8	7	6	5	4	4	3
4	4	3	2	2	1	1	0	2	3	5	7	8	9	9	8	5	3	1	0	2	5	7	9	7	7	7	6	5	5
5	5	4	3	3	2	1	0	1	3	5	7	8	9	9	8	6	4	2	1	0	1	2	8	9	9	8	8	7	8
4	5	5	4	4	2	0	2	3	4	6	7	8	9	9	7	6	5	3	2	1	0	2	5	7	9	9	9	7	7
3	5	7	5	3	1	0	1	3	5	7	8	8	9	9	7	6	5	4	3	2	0	1	3	5	6	7	9	9	8

table 3.1.1: **„landscape“, constructed from the heights of 30 columns and 20 rows**

concept 1: colour

6	6	5	5	4	4	3	2	0	0	1	2	1	0	0	2	4	6	8	9	9	8	7	5	3	1	0	0	1	2
5	5	4	4	3	3	2	1	0	1	1	2	1	0	0	1	5	7	9	8	7	6	5	3	2	0	1	1	1	2
4	4	4	3	4	3	2	1	0	1	1	2	2	1	0	3	6	9	9	8	7	4	2	0	0	1	2	2	2	3
4	4	4	5	5	4	3	2	0	1	1	2	1	0	2	4	7	9	7	5	3	1	0	0	1	2	3	3	3	4
3	4	5	6	6	5	4	3	0	2	2	1	1	0	2	4	7	9	7	4	2	0	0	1	1	2	3	4	4	5
4	5	6	6	7	7	6	6	5	0	1	2	1	0	3	5	7	9	8	6	4	2	0	1	1	2	3	4	5	6
5	6	8	8	8	8	7	8	8	6	0	1	1	0	2	5	8	9	8	7	5	3	0	1	2	3	4	5	6	7
7	8	9	9	9	9	9	9	9	9	5	0	1	1	0	4	6	9	7	6	4	2	1	0	1	2	3	4	5	6
9	9	8	8	8	9	9	8	8	9	9	4	0	0	0	3	7	9	8	6	5	3	2	0	0	1	2	3	4	5
8	8	7	6	6	7	8	8	7	8	9	9	4	2	0	4	7	9	9	7	6	5	3	1	0	1	2	3	4	5
7	6	5	5	5	6	7	7	7	8	8	9	9	5	2	0	5	7	9	8	7	5	4	2	1	0	1	2	3	3
5	4	2	3	4	3	6	5	6	5	7	8	9	9	6	2	0	2	9	9	8	8	7	5	3	1	0	1	2	2
3	1	0	1	2	1	4	4	5	4	6	7	9	9	6	3	0	0	3	9	8	7	6	5	3	2	1	0	0	0
3	2	1	0	0	1	2	2	3	4	5	7	9	9	7	4	2	0	1	4	9	7	6	6	5	4	3	1	1	1
2	2	1	0	0	0	0	2	3	3	5	7	9	9	7	4	3	2	0	5	9	9	8	7	6	5	4	3	2	2
3	3	2	1	1	1	1	0	1	2	4	6	8	9	9	8	6	4	2	0	4	9	9	8	7	6	5	4	4	3
4	4	3	2	2	1	1	0	2	3	5	7	8	9	9	8	5	3	1	0	2	5	7	9	7	7	7	6	5	5
5	5	4	3	3	2	1	0	1	3	5	7	8	9	9	8	6	4	2	1	0	1	2	8	9	9	8	8	7	8
4	5	5	4	4	2	0	2	3	4	6	7	8	9	9	7	6	5	3	2	1	0	2	5	7	9	9	9	7	7
3	5	7	5	3	1	0	1	3	5	7	8	8	9	9	7	6	5	4	3	2	0	1	3	5	6	7	9	9	8

table 3.1.2: **„landscape“, constructed from *table 3.1.1***

<u>colored</u>:

0 - 1 - 2 - 3 - 4 - 5 - 6 - 7 - 8 - 9

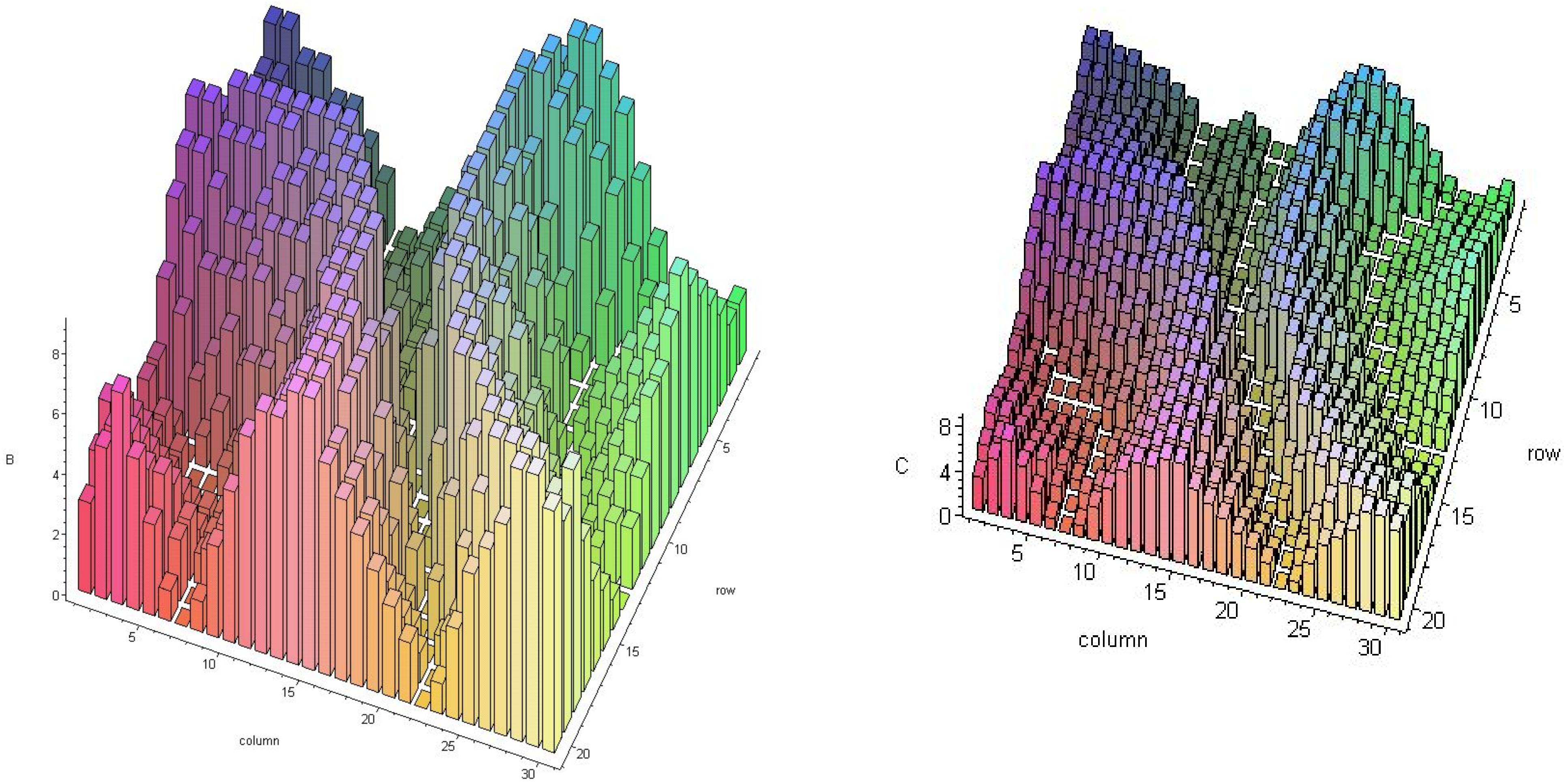


fig. 3.1.1: **„landscape 3D“, constructed from *table 3.1.1***

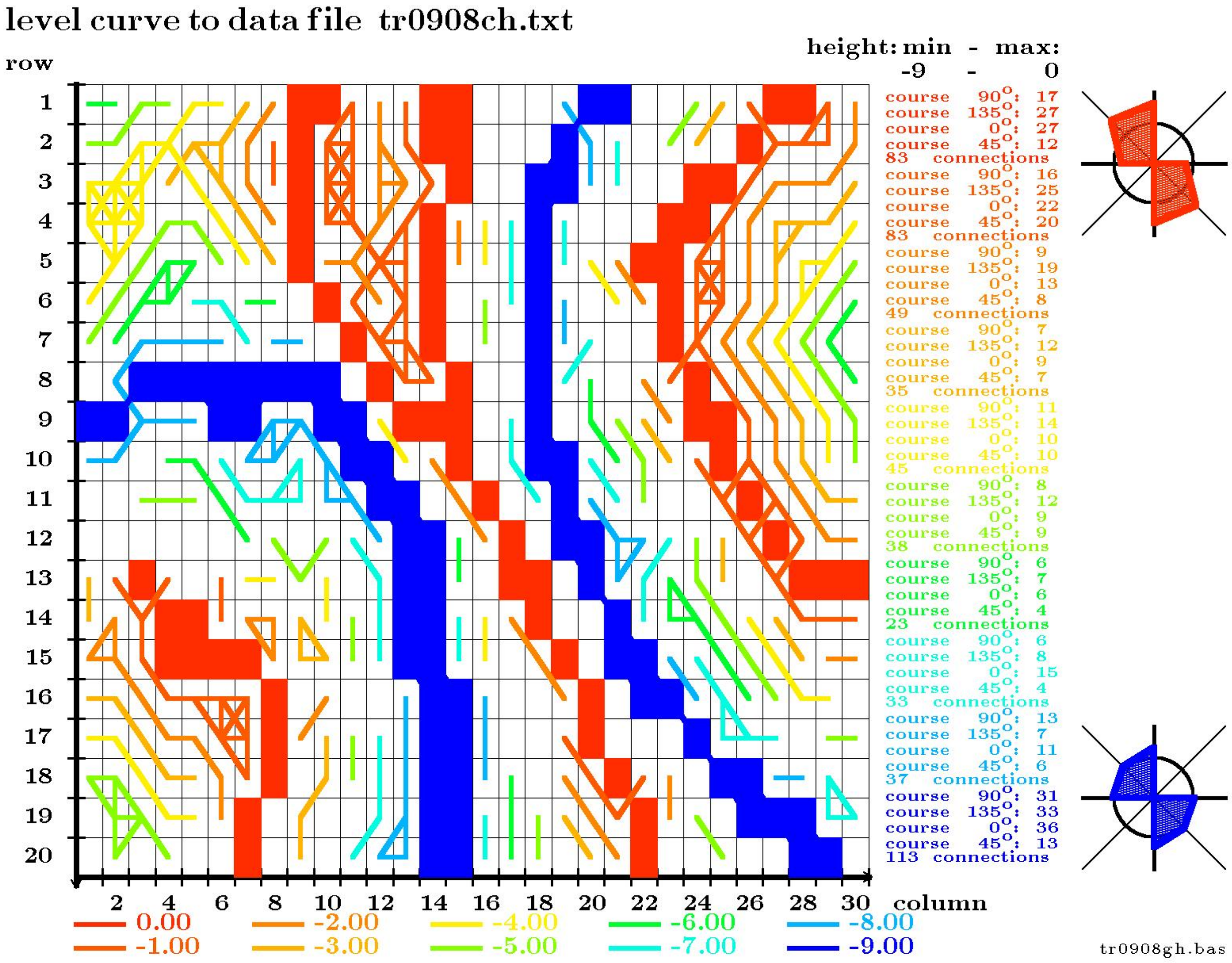


fig 3.1.2: „landscape“, constructed from ***table 3.1.1***

<u>additional:</u> lines as connection between neighbor-pixels with the same height

<u>for each class of height:</u> number of lines with the same orientation

3.2 slip bands

<u>definition 3.2.1:</u> A „<u>landscape $L(s,z)$ of voxels</u>" is a rectangle of s columns and z rows, if for each (a,b) with $1 \leq a \leq s$ and $1 \leq b \leq z$ there exists a real number $h(a,b) \in \mathbb{R}$ which we will call the „height of the pixel (a,b)".

We look at a „landscape $L(s,z)$" with hills and valleys, and we want to find slip bands in this landscape.

<u>definition 3.2.2:</u> Let $L(s,z)$ be a landscape and $h_{min} := \min\{h(a,b)\,;\, 1 \leq a \leq s$ and $1 \leq b \leq z\}$ and $h_{max} := \max\{h(a,b)\,;\, 1 \leq a \leq s$ and $1 \leq b \leq z\}$ If $h_{min} < h_{max}$ and $n_h \in \mathbb{N}$, then we define a „<u>height-class</u>"

$$T(i) := \left\{ z; h_{min} + i \cdot \frac{h_{max} - h_{min}}{n_h} \leq z \leq h_{min} + (i+1) \cdot \frac{h_{max} - h_{min}}{n_h} \right\} \qquad \textbf{for } 0 \leq i \leq n_h - 1$$

<u>definition 3.2.3:</u> Let $L(s,z)$ be a landscape and $A = (a_0, b_0) \in L(s,z)$. Then a „<u>rectangle</u>" $R_{a,b}(A)$ is defined by

$R_{a,b}(A) := \{(x,y)\,,\, a_0 \leq x \leq a_0 + a$ **and** $b_0 \leq y \leq b_0 + b\}$.

definition 3.2.4: A pixel $p = (x, y)$ in a rectangle R has the „orientation 0° with respect to R“, if there exists a number i with $0 \leq i \leq n_h - 1$ such that the pixel p and the pixel 2 (or the pixel 6) in ***fig 3.2.3*** both are in the height-class $T(i)$.

p has the „orientation 45° with respect to R“, if there exists a number i with $0 \leq i \leq n_h - 1$ such that the pixel p and the pixel 3 (or the pixel 7) in ***fig 3.2.3*** both are in the height-class $T(i)$.

p has the „orientation 90° with respect to R“,if there exists a number i with $0 \leq i \leq n_h - 1$ such that the pixel p and the pixel 4 (or the pixel 8) in ***fig 3.2.3*** both are in the height-class $T(i)$.

p has the „orientation 135° with respect to R“,if there exists a number i with $0 \leq i \leq n_h - 1$ such that the pixel p and the pixel 5 (or the pixel 9) in ***fig 3.2.3*** both are in the height-class $T(i)$.

surroundings of a voxel p

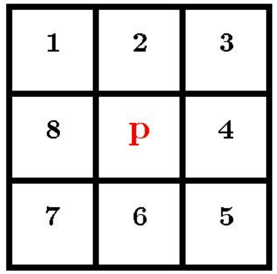


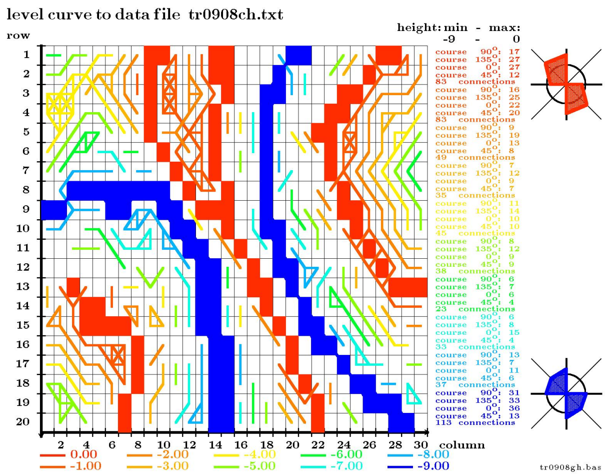
fig. 3.2.3: „orientation“

definition 3.2.5: A pixel p is „connected“ with a pixel q if the pixel q belongs to the surroundings of p in the sense of ***fig. 3.2.3***.

definition 3.2.6: Let R be a rectangle in a landscape of voxels $L(s, z)$.

Then a „slip band S_R in R“ is defined by a set of n_S (with $n_S > n_{slip}$) pixels $p \in T(0)$ which are connected. - here we will use $n_{slip} = 10$.

We want to look for slip bands in *fig 3.1.2*:
we count (on the right side) 30 pixels (blue) which are connected and are in the height-class $T(0)$ and on the left side 36 blue pixels. But a „slip band" in the sense of the material science needs a high step - and such a high step is only on the right side between the blue and 25 pixels of the red line. - There we find a high step of 9.



Now we are able to formulate our problem:

1. **From the material scientists we get a matrix with the height of 1200×2400 pixels.**
2. **The position of the microscope in relation to the sample is (always) unknown.**
3. **We have to calculate different positions of the microscope - each of this positions give another landscape $L(2400, 1200)$.**
4. **We have to choose that landscape where we can find slip bands with maximal differences in the height of valley and hill (blue and red).**

4 roughening of a surface (Sandstrahlen)

4.1 Modelling

We model the penetration of stones which strike the surface of a plate of iron. The result is a quite rough surface. For a simple camparison of different models of penetration we use such many stones, that the difference between the highest and the lowest voxel of the plate is 10 μm.

Definition: A stone unit is a stone with side length 1 and quadratic base.

Assumption 4.1:
There are three different sizes of stones:

- **size 1 (stone unit): A stone with side length 1. If a stone unit strikes the plate then 1 voxel is deepened.**
- **size 2: A stone with side length 2. The base of this stone insists on 4 stone units. If such a stone strikes the plate then 4 voxels are deepened.**
- **size 3: A stone with side length 3. The base of this stone insists on 9 stone units. which deepen the plate.**

4.2 Three different sizes

Assumption 4.2:

In our first investigation we use $\frac{n}{3}$ stones of size 1, $\frac{n}{3}$ stones of size 2 and $\frac{n}{3}$ stones of size 3, where n is the number of stones.

Assumption 4.3:

Each of these stones reaches the surface with a speed v - which is equal for all stones.

Assumption 4.4:

The position of the stone on the plate is determined by a random generator for each stone.

Assumption 4.5:

There exists a maximal penetration $p_{max} < D$, where D is the thickness of the plate

We choose $\boxed{p_{max} = 40\,\mu m.}$

Remark 4.1:
For each pixel (u, v) of a plate with s columns and z rows

$$\mathbf{O} := \left\{ (u, v) \in \mathbb{N}^2 \,,\, 0 \leq u \leq s \,,\, -z \leq v \leq 0 \right\}$$

there exists a number $n(u, v)$ of stone units which stroke this pixel.

Remark 4.2: There exists a mapping p from $\mathbb{N}$ (the number of a striking the surface) into $\mathbb{R}$:

$$p : \mathbb{N} \rightarrow \mathbb{R} \text{ ,}$$

where $p(n(u, v))$ is the penetration on pixel (u, v)

and $p_{max} := lim_{n\rightarrow\infty} p(n(u, v))$

Question 4.1:

$\boxed{\text{How does a surface look after such a „stone-brush“?}}$

4.3 linear, 685 stones, 3 different sizes

Assumption L:

$$\boxed{e(u,v) := \left(1 - \frac{1}{n(u,v)+1}\right) \cdot t_{max}}$$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	33.33	33.33	33.33	36.00	36.67	37.14	36.67	35.00	34.29	33.33	34.29	34.29	35.00	34.29	35.00	36.00
2	32.00	32.00	33.33	36.00	36.00	36.36	36.00	35.56	36.67	36.36	33.33	36.00	35.00	36.00	35.00	36.92
3	33.33	35.56	35.56	36.00	36.00	36.00	35.56	35.00	36.36	36.00	34.29	35.56	35.00	35.56	36.00	36.36
4	36.00	36.67	36.92	36.92	37.33	35.00	36.00	35.56	36.00	35.56	35.00	35.00	35.00	35.00	32.00	36.00
5	36.92	36.92	36.92	36.67	36.92	33.33	33.33	34.29	36.00	36.36	35.56	34.29	36.36	35.56	32.00	32.00
6	36.00	37.14	37.33	37.14	36.00	34.29	35.00	34.29	36.00	37.33	37.14	35.00	35.56	35.00	34.29	30.00
7	36.00	37.14	37.33	36.36	35.00	36.00	36.00	36.00	36.67	37.14	36.92	34.29	34.29	35.00	35.56	34.29
8	35.56	35.56	35.00	35.56	35.00	35.56	36.00	35.56	35.00	36.36	36.67	36.92	36.67	36.36	36.36	35.56
9	35.56	35.56	35.56	36.92	36.67	36.67	36.92	35.00	33.33	35.00	35.00	36.00	37.14	37.33	36.92	36.92
10	36.92	36.36	35.56	36.92	36.67	37.33	36.67	35.00	26.67	33.33	36.00	36.67	36.92	37.14	37.14	37.78
11	35.00	36.67	36.92	37.33	36.92	37.65	36.00	36.00	32.00	35.00	36.36	36.00	36.36	36.67	36.67	37.14
12	34.29	36.36	36.92	36.67	36.92	36.92	35.56	35.56	33.33	34.29	36.00	36.36	36.92	36.00	36.36	34.29
13	34.29	34.29	36.92	36.67	36.67	36.00	35.56	35.56	32.00	36.36	36.67	36.00	34.29	35.56	36.92	36.67
14	36.67	36.67	36.67	36.00	35.00	35.00	34.29	36.00	35.56	36.00	35.00	35.56	34.29	36.67	36.92	36.00
15	37.14	37.14	36.36	36.67	36.67	36.36	35.56	36.00	35.56	35.00	36.67	36.92	36.67	36.36	35.56	35.00
16	36.92	36.36	35.00	33.33	35.56	36.92	36.92	37.14	36.00	36.36	37.14	36.67	37.14	35.00	35.56	34.29
17	36.36	35.00	34.29	32.00	35.00	37.14	37.78	37.33	35.56	36.36	36.92	36.36	36.36	35.56	32.00	33.33
18	35.00	33.33	33.33	34.29	36.36	35.56	37.65	37.33	35.56	34.29	36.00	35.56	34.29	33.33	33.33	35.00
19	35.00	33.33	32.00	33.33	34.29	33.33	36.67	36.36	35.56	35.56	36.92	35.56	32.00	32.00	36.00	36.00
20	35.56	36.36	35.56	33.33	33.33	33.33	32.00	33.33	35.00	35.56	36.00	35.00	30.00	33.33	34.29	36.00
21	35.56	36.67	36.67	35.56	34.29	35.00	34.29	34.29	33.33	36.00	36.36	35.00	32.00	33.33	33.33	36.00
22	34.29	36.36	36.36	35.56	32.00	32.00	32.00	35.00	35.00	36.67	36.36	36.36	33.33	34.29	32.00	34.29

We have to use colors for more information

Assumption L:

$$e(u,v) := \left(1 - \frac{1}{n(u,v)+1}\right) \cdot t_{max}$$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	33.33	33.33	33.33	36.00	36.67	37.14	36.67	35.00	34.29	33.33	34.29	34.29	35.00	34.29	35.00	36.00
2	32.00	32.00	33.33	36.00	36.00	36.36	36.00	35.56	36.67	36.36	33.33	36.00	35.00	36.00	35.00	36.92
3	33.33	35.56	35.56	36.00	36.00	36.00	35.56	35.00	36.36	36.00	34.29	35.56	35.00	35.56	36.00	36.36
4	36.00	36.67	36.92	36.92	37.33	35.00	36.00	35.56	36.00	35.56	35.00	35.00	35.00	35.00	32.00	36.00
5	36.92	36.92	36.92	36.67	36.92	33.33	33.33	34.29	36.00	36.36	35.56	34.29	36.36	35.56	32.00	32.00
6	36.00	37.14	37.33	37.14	36.00	34.29	35.00	34.29	36.00	37.33	37.14	35.00	35.56	35.00	34.29	30.00
7	36.00	37.14	37.33	36.36	35.00	36.00	36.00	36.00	36.67	37.14	36.92	34.29	34.29	35.00	35.56	34.29
8	35.56	35.56	35.00	35.56	35.00	35.56	36.00	35.56	35.00	36.36	36.67	36.92	36.67	36.36	36.36	35.56
9	35.56	35.56	35.56	36.92	36.67	36.67	36.92	35.00	33.33	35.00	35.00	36.00	37.14	37.33	36.92	36.92
10	36.92	36.36	35.56	36.92	36.67	37.33	36.67	35.00	26.67	33.33	36.00	36.67	36.92	37.14	37.14	37.78
11	35.00	36.67	36.92	37.33	36.92	37.65	36.00	36.00	32.00	35.00	36.36	36.00	36.36	36.67	36.67	37.14
12	34.29	36.36	36.92	36.67	36.92	36.92	35.56	35.56	33.33	34.29	36.00	36.36	36.92	36.00	36.36	34.29
13	34.29	34.29	36.92	36.67	36.67	36.00	35.56	35.56	32.00	36.36	36.67	36.00	34.29	35.56	36.92	36.67
14	36.67	36.67	36.67	36.00	35.00	35.00	34.29	36.00	35.56	36.00	35.00	35.56	34.29	36.67	36.92	36.00
15	37.14	37.14	36.36	36.67	36.67	36.36	35.56	36.00	35.56	35.00	36.67	36.92	36.67	36.36	35.56	35.00
16	36.92	36.36	35.00	33.33	35.56	36.92	36.92	37.14	36.00	36.36	37.14	36.67	37.14	35.00	35.56	34.29
17	36.36	35.00	34.29	32.00	35.00	37.14	37.78	37.33	35.56	36.36	36.92	36.36	36.36	35.56	32.00	33.33
18	35.00	33.33	33.33	34.29	36.36	35.56	37.65	37.33	35.56	34.29	36.00	35.56	34.29	33.33	33.33	35.00
19	35.00	33.33	32.00	33.33	34.29	33.33	36.67	36.36	35.56	35.56	36.92	35.56	32.00	32.00	36.00	36.00
20	35.56	36.36	35.56	33.33	33.33	33.33	32.00	33.33	35.00	35.56	36.00	35.00	30.00	33.33	34.29	36.00
21	35.56	36.67	36.67	35.56	34.29	35.00	34.29	34.29	33.33	36.00	36.36	35.00	32.00	33.33	33.33	36.00
22	34.29	36.36	36.36	35.56	32.00	32.00	32.00	35.00	35.00	36.67	36.36	36.36	33.33	34.29	32.00	34.29

Tab. 2.3: linear, 685 stones, 3 different sizes

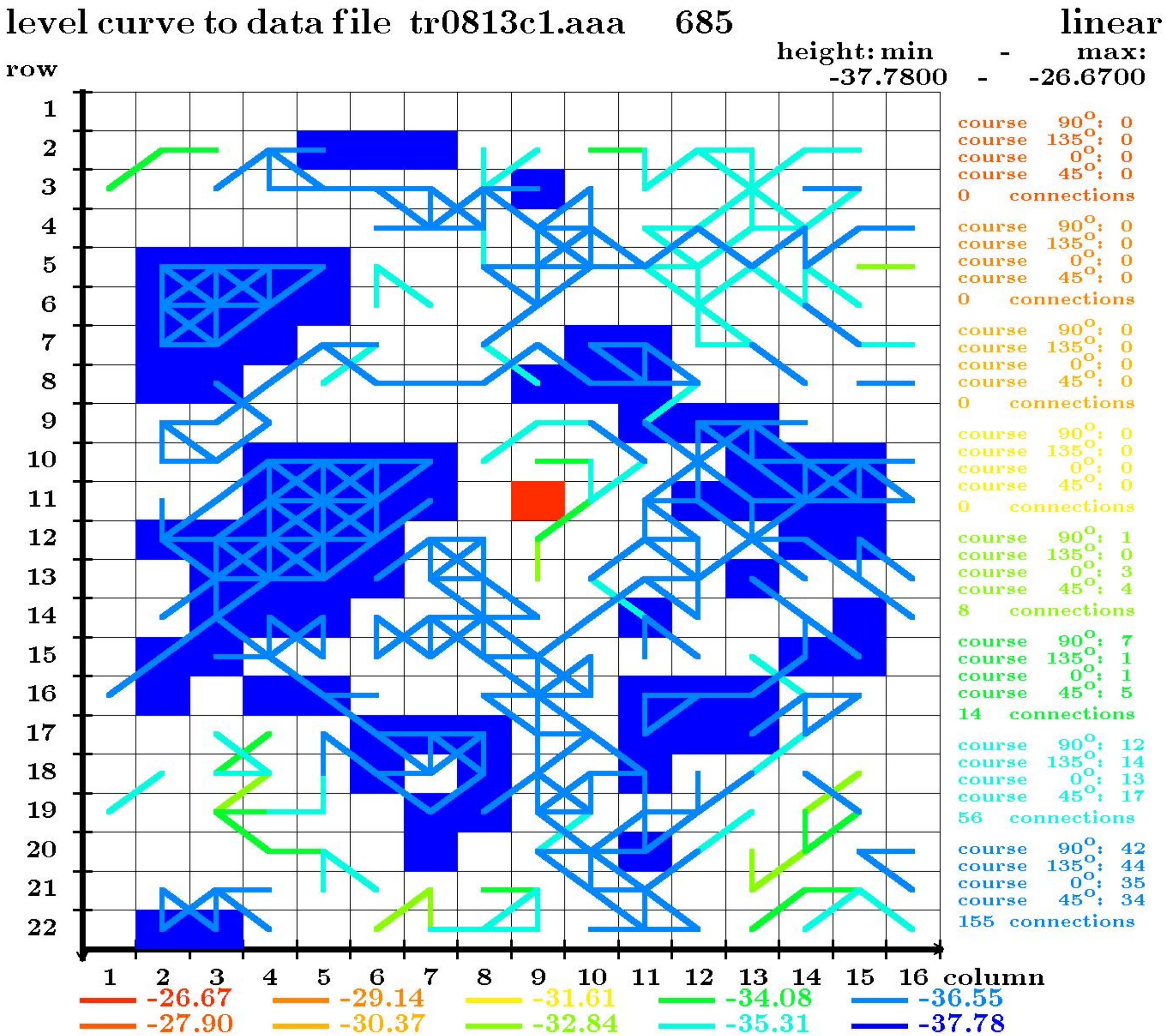


fig. 2.3: linear, 685 stones, 3 different sizes

2.4 quadratic

Assumption Q:

$$e(u,v) := \left(1 - \frac{1}{(n(u,v)+1)^2}\right) \cdot t_{max}$$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	37.50	37.50	38.40	38.89	38.89	39.18	38.89	37.50	37.50	35.56	35.56	35.56	37.50	37.50	38.40	39.18
2	37.50	37.50	38.40	38.89	35.56	35.56	38.40	38.40	39.38	39.18	37.50	38.40	38.89	39.38	38.40	39.51
3	37.50	39.38	39.38	39.38	38.40	30.00	35.56	37.50	39.38	39.18	38.89	39.38	39.38	39.38	39.18	39.38
4	39.18	39.67	39.67	39.67	39.67	37.50	38.40	37.50	39.18	38.89	38.89	39.18	39.18	39.18	38.40	39.38
5	39.67	39.67	39.60	39.51	39.51	38.40	38.40	37.50	39.18	39.38	39.18	38.89	39.60	39.38	38.40	38.40
6	39.51	39.72	39.60	39.51	39.18	38.89	38.89	37.50	39.18	39.60	39.38	38.89	38.89	38.89	38.89	35.56
7	39.51	39.72	39.60	38.89	38.40	39.18	39.38	39.18	39.51	39.67	39.51	38.40	38.40	38.89	39.18	38.40
8	38.89	38.89	38.89	39.18	38.89	39.18	39.38	38.89	38.89	39.38	39.51	39.60	39.60	39.51	39.51	39.38
9	39.18	39.18	38.40	39.51	39.51	39.60	39.51	38.89	38.40	39.18	39.38	39.18	39.51	39.60	39.67	39.67
10	39.38	39.38	38.40	39.51	39.60	39.60	39.18	38.40	35.56	38.40	39.18	39.18	38.40	39.18	39.67	39.84
11	38.40	39.51	39.51	39.72	39.60	39.80	38.89	38.89	38.40	38.40	39.18	38.89	35.56	37.50	39.18	39.72
12	38.40	39.18	39.38	39.51	39.60	39.67	38.40	38.89	38.89	38.40	38.40	38.89	38.89	38.40	38.40	38.40
13	37.50	37.50	39.60	39.51	39.60	39.18	38.89	38.89	37.50	39.51	39.18	39.18	35.56	37.50	38.40	39.18
14	39.38	39.51	39.38	39.18	39.18	38.40	35.56	38.40	38.89	39.18	38.40	38.40	35.56	39.38	39.18	39.18
15	39.67	39.67	39.18	39.51	39.38	38.89	38.40	38.40	38.40	37.50	39.18	39.18	38.89	39.51	38.40	37.50
16	39.67	39.51	38.89	38.40	38.40	39.38	39.60	39.60	38.89	39.18	39.18	38.40	39.18	37.50	38.89	37.50
17	39.38	38.89	37.50	35.56	38.89	39.60	39.72	39.67	39.18	39.18	39.51	38.40	38.89	39.18	38.40	37.50
18	37.50	38.40	38.40	38.89	39.38	38.89	39.76	39.60	38.89	38.40	38.40	37.50	35.56	38.89	38.89	39.18
19	37.50	35.56	37.50	38.40	38.89	38.40	39.38	38.89	38.89	38.40	38.40	37.50	37.50	37.50	39.51	39.51
20	38.40	38.89	38.40	38.40	37.50	38.40	38.40	37.50	38.40	35.56	35.56	37.50	35.56	38.40	38.89	39.38
21	35.56	38.89	39.18	38.89	35.56	38.40	39.18	38.89	35.56	37.50	30.00	35.56	35.56	38.40	37.50	39.18
22	37.50	39.38	39.51	39.18	35.56	35.56	37.50	39.18	38.89	39.18	37.50	38.89	35.56	38.40	30.00	37.50

Tab. 2.4: quadratic, 450 stones, 3 different sizes

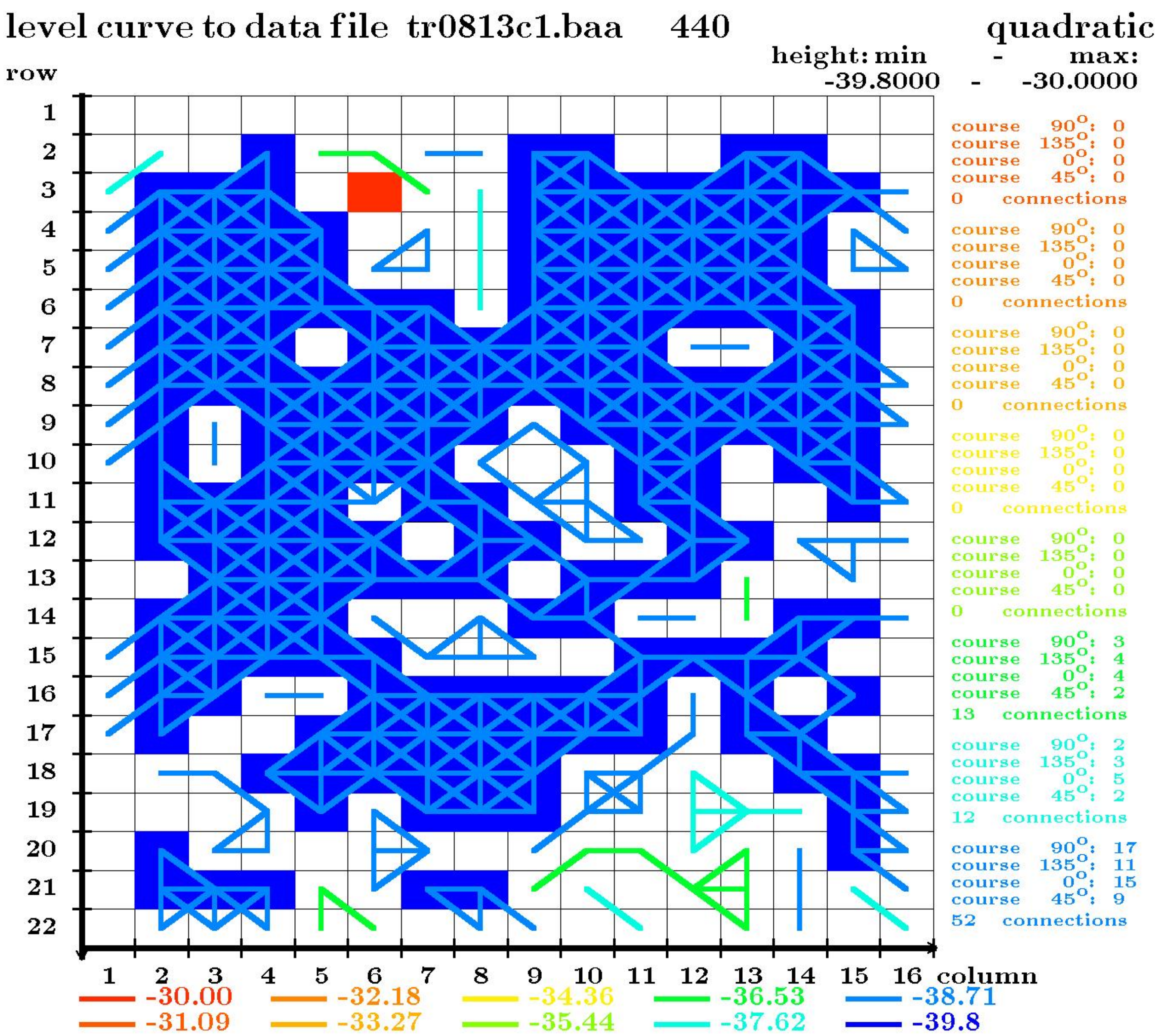


fig. 2.4: quadratic, 440 stones, 3 different sizes

2.5 root

Assumption R:

$$e(u,v) := \left(1 - \frac{1}{\sqrt{n(u,v)+1}}\right) \cdot t_{max}$$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	27.94	28.45	28.91	29.31	30.00	30.82	29.67	27.35	26.67	27.35	25.86	25.86	26.67	26.67	28.91	30.00
2	26.67	27.94	29.67	28.91	28.91	29.67	29.31	28.91	30.00	30.30	26.67	28.91	27.35	28.45	25.86	30.00
3	26.67	28.91	30.00	28.91	28.91	29.67	28.91	28.45	29.31	28.91	28.45	29.31	28.45	28.45	28.45	29.67
4	28.45	29.67	30.57	30.57	31.47	29.31	29.67	29.31	28.45	27.35	28.45	28.45	28.91	27.94	24.88	28.91
5	29.31	30.00	30.30	30.57	31.47	27.94	26.67	26.67	27.35	27.94	26.67	27.35	29.67	27.94	22.11	23.67
6	27.94	30.57	30.57	30.57	28.45	26.67	26.67	26.67	27.94	30.00	30.30	28.91	29.67	27.94	24.88	20.00
7	28.45	30.30	30.57	29.31	27.35	28.91	27.94	28.45	29.31	29.67	30.30	28.91	28.91	27.35	27.35	26.67
8	28.91	28.45	27.35	27.94	26.67	27.94	28.91	28.91	28.45	29.31	29.31	30.30	30.57	29.31	29.31	28.45
9	29.31	28.45	27.35	29.67	29.67	30.00	30.57	28.91	27.35	28.45	27.94	28.45	30.57	30.82	30.30	29.31
10	30.57	30.00	27.94	30.30	30.30	31.27	30.30	27.94	24.88	28.45	28.91	30.00	30.57	30.82	30.57	31.47
11	27.35	29.67	30.30	30.82	30.30	31.66	30.30	29.67	25.86	28.45	29.31	28.91	28.45	30.57	30.30	30.82
12	25.86	28.45	29.67	30.30	30.82	30.82	29.31	29.31	27.94	27.94	28.91	28.45	29.31	28.45	28.91	27.94
13	25.86	26.67	29.67	30.57	30.00	28.91	29.31	29.31	27.35	28.91	29.31	29.31	27.94	28.45	30.00	30.82
14	28.91	29.67	30.00	28.45	28.45	28.45	27.35	28.45	28.45	28.45	27.35	28.91	27.94	30.30	30.00	30.00
15	29.31	29.67	28.91	29.67	29.31	28.91	27.94	28.91	28.91	26.67	29.31	30.00	30.57	30.57	28.91	29.67
16	30.00	29.31	27.94	26.67	27.35	30.00	30.57	31.06	30.30	29.67	30.82	30.00	30.30	28.91	29.31	29.67
17	29.31	27.94	26.67	25.86	27.94	30.57	31.27	31.06	29.31	30.00	30.57	30.00	29.31	27.94	25.86	27.94
18	27.94	27.35	27.35	27.94	29.67	28.45	31.06	31.06	28.91	27.35	28.91	28.45	25.86	23.67	24.88	28.45
19	27.35	27.35	27.94	26.67	28.45	28.91	30.30	30.00	28.45	28.45	30.00	28.45	23.67	23.67	27.94	27.94
20	28.45	30.00	30.57	26.67	27.35	26.67	27.35	28.45	28.45	29.67	30.00	28.91	22.11	24.88	25.86	28.45
21	27.94	29.31	29.67	28.45	27.35	27.94	28.45	27.94	27.94	29.31	30.57	29.31	27.35	28.45	27.35	29.31
22	27.35	29.31	29.31	29.67	28.45	26.67	24.88	27.94	29.31	29.67	29.67	29.67	27.35	28.45	27.35	27.94

Tab. 2.5: root, 980 stones, 3 different sizes

min=-31.66 max=-20.00 mittlere Tiefe=-28.67 mittlere Füllhöhe (= mittlere Tiefe - min =) 2.99

Mittlere Rauheit: 1.04 Mittlere quadratische Abweichung von der mittleren Füllhöhe: 2.23

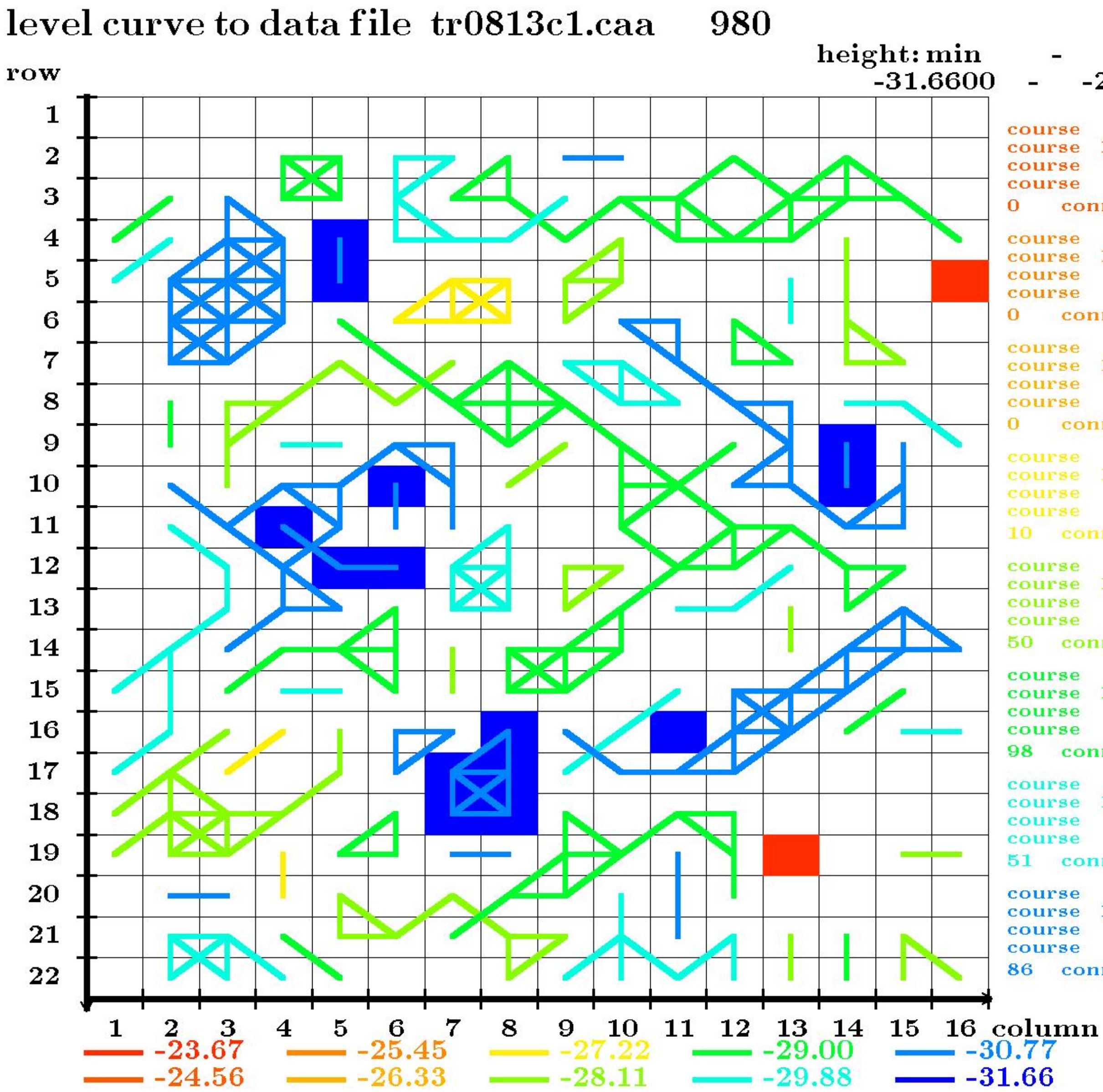


fig. 2.5: root, 980 stones, 3 different sizes

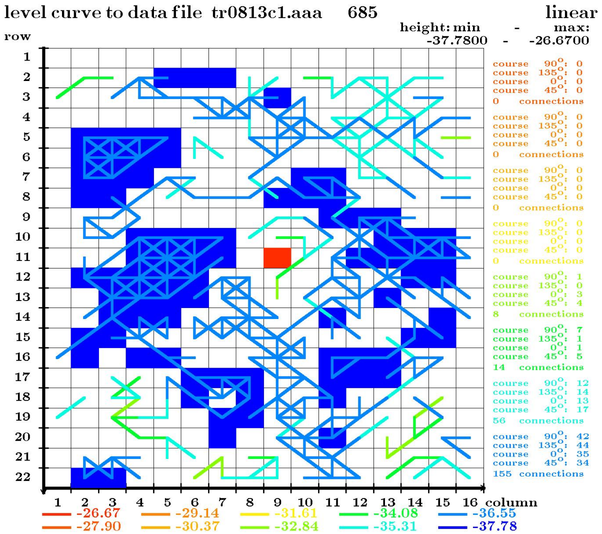


fig. 2.3:
linear, 685 stones, 3 different sizes

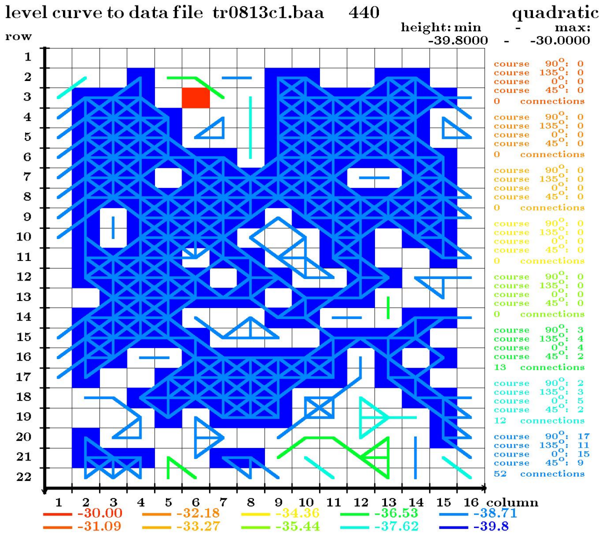


fig. 2.4:
quadratic, 440 stones, 3 different sizes

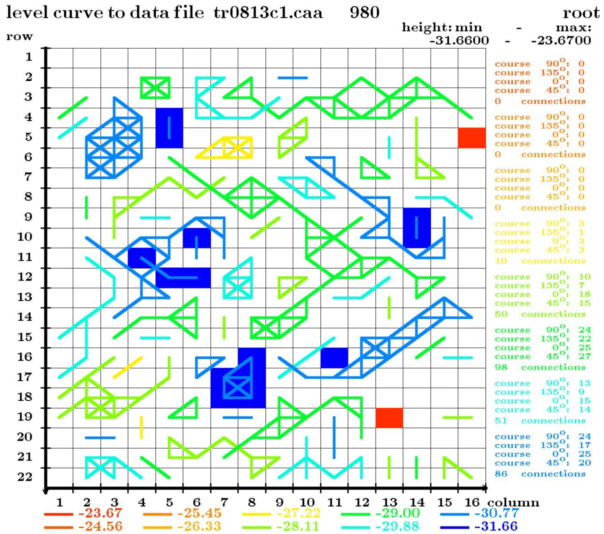


fig. 2.5:
root, 980 stones, 3 different sizes

Of course we have to look for other combinations:

1. **what happens if we use double of stones?**

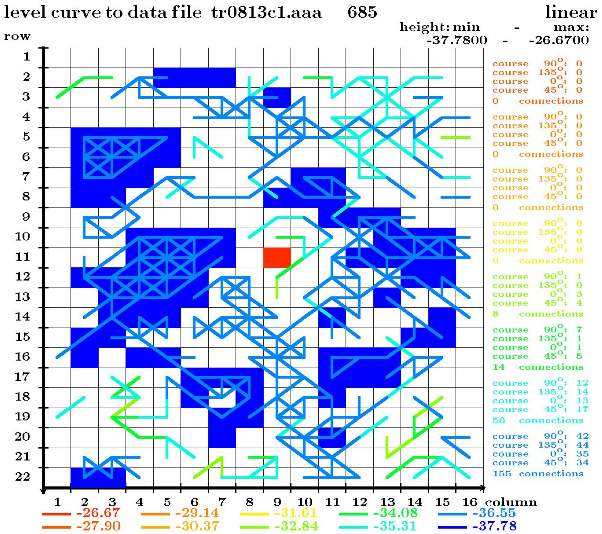


fig. 2.3:
linear, 685 stones, 3 different sizes

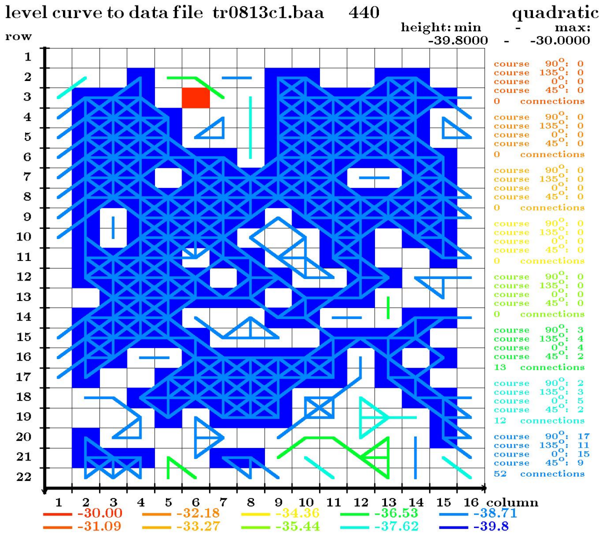


fig. 2.4:
quadratic, 440 stones, 3 different sizes

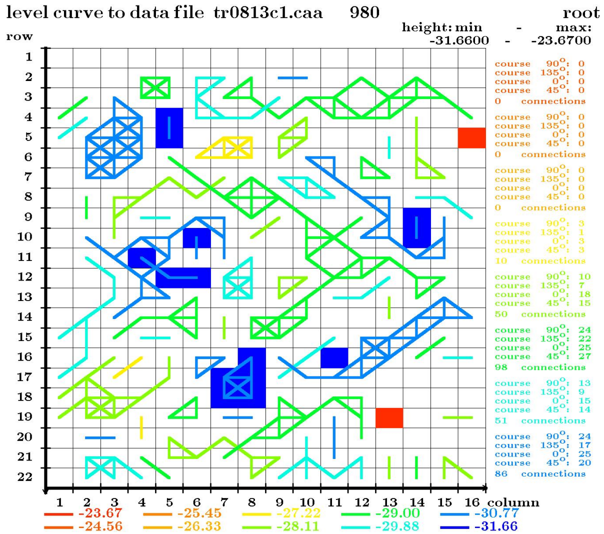


fig. 2.5:
root, 980 stones, 3 different sizes

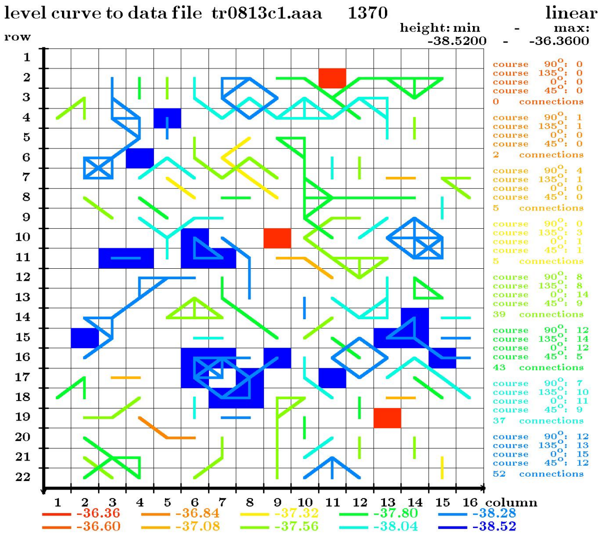


fig. 2.6:
linear, 1370 stones, 3 different sizes

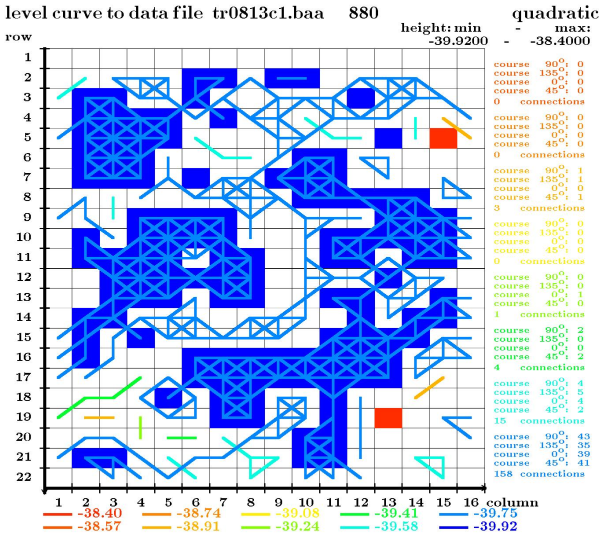


fig. 2.7:
quadratic, 880 stones, 3 different sizes

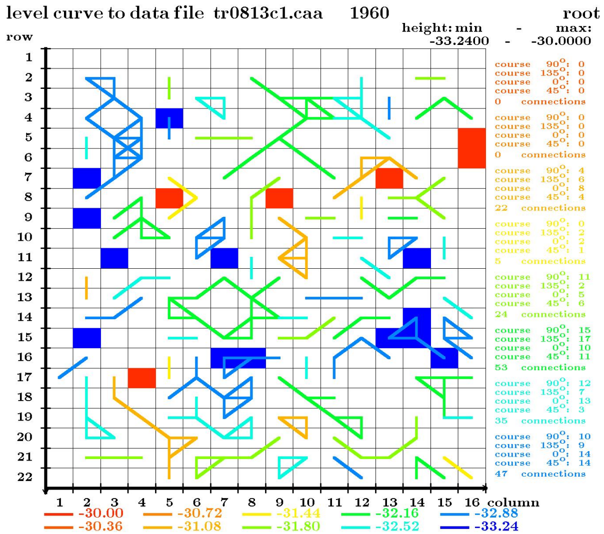


fig. 2.8:
root, 1960 stones, 3 different sizes

And we have to look for other combinations:

2. **what happens if we use other combinations of**
   stones of size 1, stones of size 2 and stones of size 3
   for instance: such that the weight of the stones of size 1 is equal to the weight of the stones of the other sizes?
3. **what happens if we use stones of only one size?**
4. **(general:) what happens if n stones impact a metal at only one point (what is the depth of penetration of n stones)?**

In the first part of this investigation we constructed roughened surfaces.

In a second part we need measurements of a real roughened surface.

These measurements should give the possibility to compare

- **the „pictures“ with its interpretations which we constructed before**

  with

- **the real data**

May be we find an affinity between reality and a geometrical description - and therefore also an idea for a mathematical description of the penetration.

**Perhaps you think that I use worse definitions-
and perhaps you are right.**

Sometimes it is very necessary and very interesting to work together with other scientific people: there are a lot of unsolved geometrical problems[2])

Therefore:

Thank you for listening

[2]) But there are extremely few possibilities for publication.